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Abstract
Rogue waves are abnormally high, with respect to the mean, waves in the ocean. Present understanding of their nature will be reviewed and discussed. Rogue waves can be due to quasi-linear superpositions of waves and nonlinear effects such as instabilities of wave trains. Both appear to be important and possible. Individual waves can be focused into a superposition due to either dispersive or directional features of wave fields. While probability of the former in oceanic conditions is very low, the directional focusing appears to be rare but regular events. Nonlinear wave fields should be separated into stable and unstable conditions, with different probability distributions for wave heights/crests. In stable conditions, wave statistics are determined by the quasi-linear focusing, whereas in unstable wave trains high transient wave events can occur. Their maximal height/steepness is determined by combined dynamics of the instability growth and the limiting wave breaking.

Keywords: Rogue Waves, Wave Focusing, Modulational Instability, Wave Breaking.

1. INTRODUCTION

Rogue, or as they are also often called freak waves have captured imagination of general public, scientists, sailors and ocean engineers for a long time. Once believed to be a legend, their existence has been definitely confirmed over the last few decades. While everybody assumes these to be abnormally high waves (or an abnormally high wave) with respect to significant wave height H_s, exact definitions for the rogue waves still vary. In oceanography, these range from a rogue wave being twice H_s to $2.2H_s$. The former individual event can be expected within a long enough storm as a result of linear superposition of waves, whereas the latter cannot, and yet they do occur and are documented. Therefore, while there is a substantial and successful effort in the community to update physics of the wave-forecast models (e.g. Rogers et al., 2012), this is still an open question: how such individual extreme events can be predicted within the spectral approaches.

In ocean engineering, additional conditions are sometimes imposed on how high the wave crest η is with respect to the wave height H of the rogue wave. Such definition is driven by industry design criteria, but is apparently less general. Early anecdotal evidences of rogue waves broadly subdivided them into three categories: "wall of the water" which would indeed be a very high crest, "hole in the sea" which is obviously a very deep trough, and "three sisters" which is a sequence of rogues waves or a rogue wave group. All three have now been observed and instrumentally recorded and, for shipping, for example, the deep trough can be even more dangerous than the preceding high crest.

Therefore a more general definition should rely on wave height, but even then there is an ambiguity on which trough is selected to define the trough-to-crest height, particularly in three-dimensional environments of directional wave fields where each crest is surrounded by a non-uniform trough (Fig.1). Another relevant property, from the point of view of potential impact of a rogue wave, can be

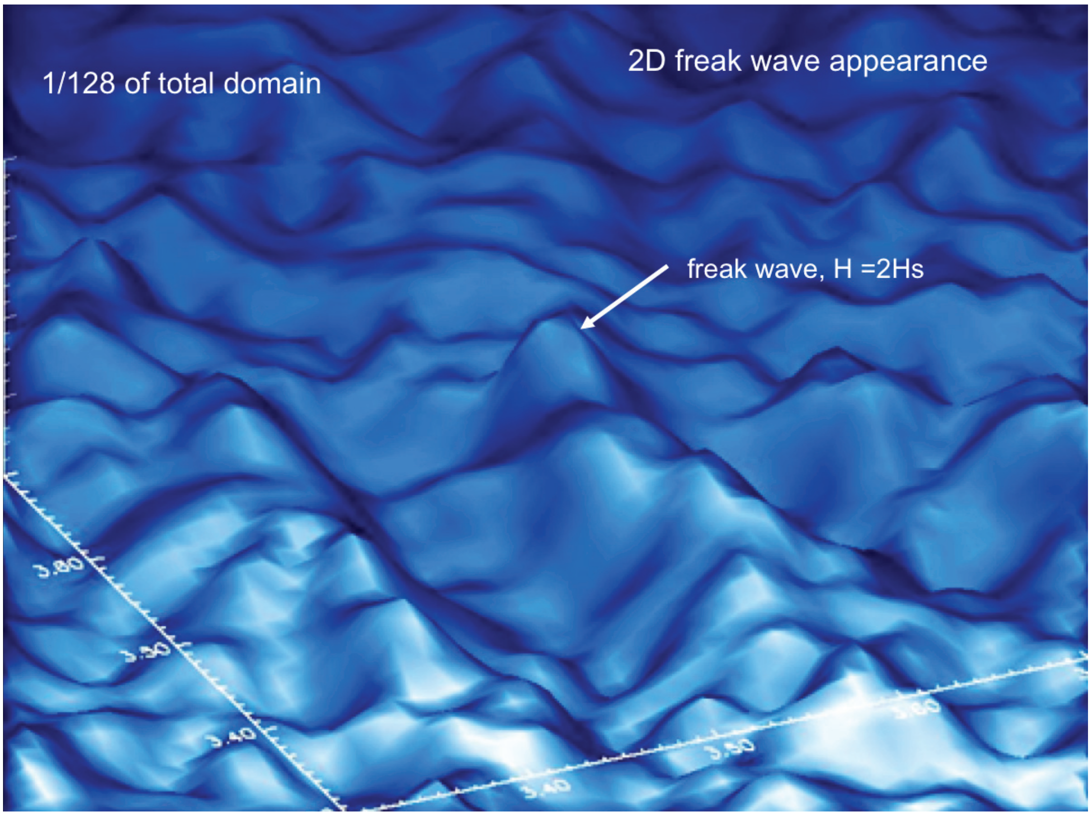

Figure 1. Freak wave produced in three-dimensional wave model (Chalikov and Babanin, 2013)

its steepness. A very high, but long and gently-sloped wave would not present a navigational hazard, but would be very dangerous for a stationary offshore rig if its crest is higher than the elevation of lower deck.

Here, we will not rely on definitions for freak waves, as we will be discussing conditions where individual waves of abnormal heights can occur, rather than analysing the rogue wave events per se. In this regard, for example, Gemmrich and Garrett (2008) argued that even a $1.9H_s$ wave is very dangerous because it is unexpected. Relative-to-the-mean nature of such definition, however, means that from the oceanographic perspective a 40 cm wave in a 20 cm-high mean wave field, while not dangerous at all, is still a rogue event. Dynamics of gravity-driven freak waves scaled this way is equivalent, regardless their physical size. This obvious fact should be kept in mind, for scientist and for sea goers the problem and definition of the rogue waves will never be the same.

Height of ocean surface waves can be strongly influenced as a result of modulation of shorter waves by longer waves, by surface currents with horizontal velocity gradients, by bottom proximity. For general summaries of the rogue wave problem we refer the reader to available review papers and books (e.g. Kharif et al., 2009). Here, we will concentrate on extreme events in most basic conditions, i.e. those among deep-water wind-forced dominant waves (waves at the scale of the spectrum peak). This is the simplest situation for oceanic waves, and yet complex enough to evade convincing and straightforward description for years. Once the simplest condition is understood, it can be used as a benchmark to approach more complicated circumstances mentioned above.

In deep water, abnormal elevations of water surface can be perceived due to two main physical reasons. We could call respective analyses as steady-wave and transient-wave approaches. The first one relies superposition of a number of individual waves due to dispersive and directional nature of surface water waves in the ocean. Such superposition approach to extreme waves has dominated the ocean-engineering applications where a lot of effort has been dedicated to developing probability distributions for wave heights, crests and troughs. We will discuss this approach in the next section.

The probability distributions are not intended and usually cannot capture individual transient wave events, when a wave looses its steady shape and transitions to a different shape. Here, we do not mean slow or linear variations of the shape because of, for example, gradual growth due to wind forcing or passage of a wave through the group envelope. As an example of a strongly nonlinear transient wave event we can point to a wave breaking when a wave may loose substantial part of its height or even disappear completely within a fraction of wave period (e.g. Babanin, 2011a). What is relevant in the context of generation of extreme wave heights is growth of individual waves or wave groups due to instabilities of nonlinear wave trains and fields such as modulational instability (Zakharov, 1966, 1967, Benjamin and Feir, 1967, McLein, 1982). Such growth can be sudden and occurs at the scale of one wave period (Chalikov, 2009). Nature of such transient individual wave events is different from the high surface elevations due to superposition of a number of steady waves. The modulational-instability to freak waves has been prevalent in the fluid mechanics community over the last decade or so. Transient rogue waves will be discussed in Section 3.

In both communities and both approaches, it is typical to consider the wave evolution, whether linear or nonlinear, as if there is no wind forcing and wave breaking. While this is a valid fluid mechanics problem, ocean waves are wind-generated and they break if are steep enough. Swells propagate away from the storm areas and are usually not subject to local winds and breaking, but then they are lower-energy seas and do not present interest in the rogue-wave context. The highest seas, however, are always wind-driven. Winds, if they are strong, can blow away wave crests and reduce the wave-breaking steepness (e.g. Babanin et al., 2010), and therefore impact even the linear superposition of waves. For the modulational instability, however, influence of the winds can be essential no matter how strong the wind is (Trulsen and Dysthe, 1992, Waseda and Tulin, 1999, Galchenko et al., 2012, Onorato and Proment, 2012). Extreme wind forcing, such as the hurricane conditions, changes the entire dynamics of surface waves and air sea interactions, may alter the growth rate, parameter range or even suppress the wave instability (e.g. Babanin, 2011b). In terms of freak wave occurrence and their probability such conditions should be considered separately. The wind-forcing issues are outlined in Section 4.

Role of the wave breaking in understanding and predicting the rogue waves is discussed in Section 5. It is argued that it is as important as the role of the mechanisms which trigger the rapid wave growth. No matter how significant these mechanisms, the waves do not grow unlimited and a physically possible wave height is limited by the wave breaking (Babanin et al., 2007, 2010, Babanin, 2013). The last Section summarises the conclusions.

2. SUPERPOSITION OF WAVES

The wave superposition approach is often regarded as linear or quasi-linear, as opposed to the nonlinear evolution of wave groups due to modulational instability. Nothing, however, can be classified in the dynamics and physics of ocean waves in simple terms.

The superposition occurs for three reasons. First, waves of different scales propagate with different speeds. The ocean wave spectrum is continuous, therefore waves of all scales within the spectrum are present and propagate at the same and in a random wave field can intersect and pile up into a very high surface elevation. This is the most commonly perceived cause for the wave superposition. Depending on whether linear (sinusoidal) or nonlineary-shaped (Stokes) waves are superposed, probability distributions for extreme wave heights, and particularly for wave crests and troughs are essentially different (e.g. Young, 1999, Forristal, 2000, Toffoli et al., 2008).

The second reason for wave dispersion and possible superposition is purely nonlinear. Waves of the same scale propagate with different speeds depending on their steepness. Maximal phase-speed difference due to such correction is small, about 5%, but even this makes waves of similar frequency more likely to intersect, and so high and breaking waves are produced more often (e.g. Pierson et al., 1992).

In directional wave fields even waves with the same frequency and same steepness can be focused and superposed if they come from different angles (e.g. Fochesato et al., 2007). This brings us to the third reason, directional. While the focusing in the first and the last case is apparently linear, last stages of the focused-wave dynamics demonstrate essentially nonlinear behaviours if the steepness becomes large enough (Brown and Jensen, 2001).

Which superposition, if any, can lead potentially to freak waves in the ocean? The dispersive superpositions, which are implicitly the backbone of the probability distribution approaches, can lead to a $2H_s$ as mentioned above. To provide statistically feasible conditions for waves higher than that, storms would need to be unrealistically long, i.e. last for weeks or even months (see e.g. Holthuijsen, 2007).

Babanin et al. (2011a) specifically investigated whether wave breaking occurs due to wave superposition or modulational instability in a directional wave tank. The breaking occurs when a wave becomes too high, reaches limiting steepness and as a result the water surface collapses, i.e. this is essentially a high-wave-occurrence problem as will be further discussed below. They argued that in a wave field with continuous spectrum, dispersive superpositions can happen often, but the problem is that waves with different speeds have also very different amplitudes. As an approximate rule, if the Phillips spectrum of f^{-5} is used as a scale, then if frequency f is doubled (i.e. phase speed is decreased two times), the amplitude a on average will drop by some $\sqrt{32} \approx 5.7$ times. With the typical mean steepness of ocean waves of ak~0.1, this would require superposition of a great number of individual waves in order to make a dominant wave reach limiting wave-breaking steepness ak~0.45 (Babanin et al., 2007, Toffoli et al., 2010) or even ak~0.2 (Chalikov and Babanin, 2012), which then appears a low-probability event. Here, k is wavenumber.

The focusing due to directionality, however, was found to be a regular cause leading to wave breaking in Babanin et al. (2011a). If waves of the same scale come from different directions, then a superposition of only two waves is needed to double wave height and steepness. Therefore, sea conditions when two or more wave systems of approximately the same peak frequency coexist are particularly dangerous. They can both produce regular events with the height being summation of the two wave heights (Greenslade, 2001, Babanin et al., 2011b) and, at certain angles, some mechanisms of wave instability are activated (Alber, 1978, Cavaleri et al., 2012).

Linear superposition of waves is most likely at small angles (because even short-crested waves stay longer in the superposed stage once it happens) and at angles close to 180 degrees. The latter is particularly dangerous for sea going even at relatively low significant wave height (Greenslade, 2001), but in case of standing waves this can lead to much larger heights of superposed waves because the limiting breaking steepness of ak~0.6-0.7 for such waves is about 50% higher than that for progressive waves (e.g. Schwatz and Fenton, 1982, Babanin, 2011). At angles between 40-60 degrees, nonlinear-instability effects are most prominent as was shown by Cavaleri et al. (2012) by means of simulations with two coupled Nonlinear Schrodinger Equations. This effect drops at angles greater than 70 degrees, but cross-seas close to 90 degrees are most punishing for shipping irrespective of wave steepness or freak waves. As a result, regardless of the actual direction of the waves with respect to each other, mean wave height, mean wave steepness and probability of freak wave occurrence, directional seas are all potentially hazardous.

3. MODULATIONAL INSTABILITY

The importance of modulational instability with respect to extreme waves was reviewed in detailed by Babanin (2012). Here we will follow this paper and provide highlights of important issues.

If the wave fields are stable, then probability of wave crests should be well described through nonlinear approximations to the expected shape of Stokes waves, such as the second-order wave crest distribution of Forristal (2000). It should be noted that even at the second-order approximation the directionality of real wave fields complicates the conclusions and outcomes (Forristal, 2000, Toffoli et al., 2007). The third-order effects, however, apart from further nonlinear corrections to the wave shape, bring about dynamic nonlinear exchanges and instabilities. The instabilities can lead to rare transient events of high waves. Because they are transient, they will not be depicted through the geometric quasistationary approach. Because they are rare, they will not be uncovered in observations or simulations (unless the simulations specifically target such events), and cannot be obtained as a result of extrapolations of the bulk observations/modelling towards the low-probability tail of the wave crest/height distributions.

These are, however, the events that can produce rogue waves unrelated to the superposition/focusing of linear/nonlinear waves. Therefore, it is important to understand first, which nonlinear wave trains and fields are stable and which are unstable. The former can be treated in terms of the superposition approach, but for the latter the approach should be different.

3.1 Instability of unidirectional wave trains

For monochromatic one-dimensional wave trains, instability has been studied extensively over the years and its behavior is well understood. Such trains are unstable to perturbations, i.e. presence of oscillations of a different frequency, even if magnitude of the perturbations is very weak. Wave frequency of the perturbations relative to the frequency of the original wave, and steepness of the original wave should be, however, within some instability range.

Diagrams of the instability range go back to the original paper by Benjamin and Feir (1967) and we can also refer the reader to a comprehensive review by Yuen and Lake (1982). The original research, however, was done for waves of small steepness, and here in Fig.2 we show a diagram reproduced from Tulin and Waseda (1999) which also includes the instability range of steeper waves.

In Fig.2, vertical scale is the growth rate of instability β_x normalised by the initial steepness of uniform monochromatic waves $\varepsilon = a_0 k_0 = ak$; a_0, k_0 and $\omega = \omega_0$ are amplitude, wavenumber and radian frequency of the carrier wave, respectively. $\delta\omega$ defines frequencies of the perturbations, to which the original wave train is unstable, i.e. $\omega = \omega_0 \pm \delta\omega$.

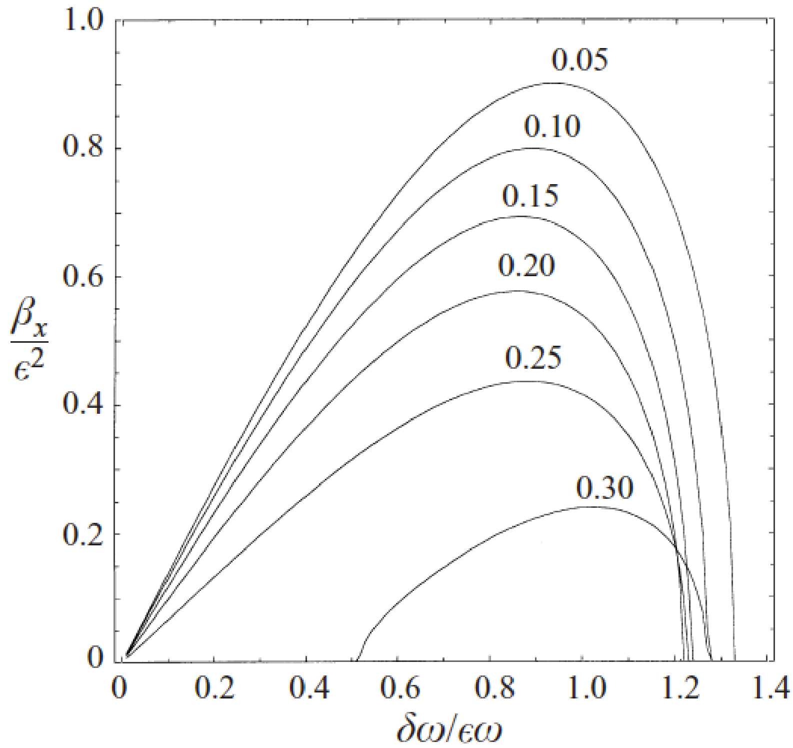

Figure 2. Initial growth rate, β_x, normalised by the initial steepness of wave train ε^2, for steepness ε = 0.05, 0.10, 0.15, 0.20, 0.25 and 0.30, plotted versus normalised modulational frequency $\delta\omega/\varepsilon\omega$. Figure is reproduced from Fig. 1 of Tulin and Waseda (1999) and is copyright of the Cambridge University Press.

Note the significance of combined parameter $\delta\omega/\varepsilon\omega$ which is the horizontal scale in the graph. This parameter defines whether the train is stable or unstable subject to the perturbations. For waves of steepness ε =0.05, for example, the instability range is from $\delta\omega/\varepsilon\omega$ =0 to ~1.33, with the maximum growth occurring at $\delta\omega/\varepsilon\omega \approx 1$, i.e. for $\delta\omega/\omega \approx 0.05$. For steeper waves, the instability range shrinks. In this paper, we will use the inverse version of the instability parameter, which is sometimes called Benjamin-Feir Index (*BFI*, e.g. Janssen (2003)). We will call it Modulational Index (M_I) both for convenience of notation and in order to distinguish its general form from the quasi-linear Benjamin-Feir theory:

$$M_I = \frac{a_0 k_0}{\delta\omega / \omega} \cdot \tag{1}$$

3.2 One-dimensional wave trains with full spectrum

One-dimensional wave trains with continuous wave spectrum have academic rather than practical interest. The wind-forced waves with continuous spectrum, are always directional (e.g. Babanin and Soloviev, 1998). Discussion of such one-dimensional full-spectrum academic case, however, is helpful as an intermediate step in the argument leading from the quasi-monochromatic one-dimensional wave trains to the two-dimensional surfaces with the full frequency-directional wave spectrum.

It should be stressed that there is no strict analytical theory for instability of waves with continuous spectrum. Credit has to be given to Onorato et al. (2001) who introduced an analogue of Benjamin-Feir Index (1) for continuous wave spectra, where some characteristic mean steepness was used instead of ε and a relative width of the wave spectrum instead of $\delta\omega/\omega$. Janssen (2003) conducted a detailed study of instability of such wave spectra by means of the Zakharov equation and concluded that $BFI \sim 1$ signals a transition from stable (BFI≤1) to unstable (BFI≥1) wave fields.

Ribal et al. (2013) investigated evolution of nonlinear wave trains with full spectrum, both unidirectional and directional, by means of the Alber equation. Overall, their conclusions agree with the argument of Onorato et al. (2001) and Janssen (2003), but they further pointed out that, unlike in the monochromatic wave trains, steepness and bandwidth for spectral waves are not unambiguous properties. For most frequently used parameterisation of wind-wave spectra, JONSWAP spectrum (Hasselmann et al., 1973), the same steepness can be achieved either by varying the equilibrium level of the spectrum a or the peak enhancement factor γ.

The two parameters, however, imply different physics with respect to the instability. If γ is fixed and the steepness is increased by raising the total spectrum energy level α, such variation makes the spectrum broader and can slow down or even prevent the instability. If, on the contrary, α is fixed and γ is raised to increase the steepness, the effect is opposite because the spectral peak becomes narrower which is what matters for instigation of the instability. Thus, based on their numerical simulations, Ribal et al. (2013) introduced a new dimensionless property which describes transition to the conditions of instability in case of JONSWAP spectrum:

$$\Pi_1 = \frac{\varepsilon}{\alpha\gamma} < 1 \cdot \tag{2}$$

Before applying this equation, note that there may be a factor of $\sqrt{2}$ difference between various definitions for the wave steepness mentioned throughout the current paper.

3.3 Instability of directional wave fields

Instability of directional wave fields is of key importance for the topic of rogue waves. First of all, real ocean wind-forced waves are always directional. Second, it has been recognised very early that the modulational instability is impaired or even suppressed in directional wave fields (McLean, 1982).

Therefore, there is a question, whose answer is still pending to a great extent, whether the real ocean wave fields are too broad or are still narrow enough for the instability to be present in such wave field. Qualitatively, Onorato et al. (2002, 2009a,b), Waseda et al. (2009a,b) demonstrated that, for typical JONSWAP frequency spectra, the modulational instability does depend on the directional spread. Quantitatively, we will outline the answer to this question in terms of the directional spread parameter A introduced by Babanin and Soloviev (1987, 1998):

$$A(f)^{-1} = \int_{-\pi}^{\pi} K(f,\phi)\,d\phi, \qquad (3)$$

where φ is the wave direction, $K(f,\phi)$ is the normalised directional spectrum:

$$K(f,\phi_{\max}) = 1, \qquad (4)$$

i.e. higher values of A correspond to narrower directional distributions.

Babanin et al. (2010) suggested a directional analogue of M_I as Directional Modulational Index

$$M_{Id} = A \cdot ak \qquad (5)$$

which is effectively a ratio of steepness and normalised directional bandwidth A^{-1} and which would signify whether the modulational instability takes place or not in the directional wave fields. That is, if the directional spreading broadens, this can be compensated by increasing the characteristic steepness, and vice versa.

Babanin et al. (2011a) conducted a laboratory experiment in a directional wave tank in order to answer the quantitative question of whether the modulational instability is still active in directional wave trains with typical angular spreads and typical mean steepness of those of the ocean waves. In the frequency/wavenumber space the wave trains were quasi-monochromatic, but their directional spectrum varied. They found the modulational instability limit as

$$A^{*}ak \approx 0.18. \qquad [6]$$

That is, for $M_{Id} \geq 0.18$ directional wave trains are unstable. Such limit is quite feasible. According to Babanin and Soloviev (1998), for dominant waves in realistic directional wind seas, A = 0.8-1.8. Therefore, with A~1 there should be ak~0.2 which is possible, and for A~1.8, there should be ak~0.11 which is typical steepness of ocean waves.

For the full frequency-directional spectrum, Mori et al. (2011) conducted a set of Monte Carlo simulations of the Nonlinear Schrodinger Equation in two horizontal dimensions, in order to investigate the modulational instability. They proposed an alternative version of BFI, where bandwidth increases at expense of an additional term which depends on the directional spectral width.

Ribal et al. (2013), like with the one-dimensional spectrum as described in Section 3.2, argued that for directional waves with continuous spectrum the criterion for transition from stable to unstable seas has to account for the spectral shape rather than mean steepness alone. For JONSWAP spectrum, their criterion is

$$\Pi_2 = \frac{\varepsilon}{\alpha\gamma} + \frac{\beta}{\varepsilon A} < 1.1 \qquad (7)$$

where β = 0.00256 is an empirical parameter.

Overall, problem of directionality of ocean waves is much more complicated than outlined here. Apart from further issues related to the directional spreading of wave energy, it also connects to three-dimensional structure of wave crests. We refer the reader to Babanin (2013) for more details.

4. WIND FORCING

In Sections 2 and 3, we discussed physical reasons responsible for sudden growth of surface elevation and/or the wave height. In the next two Sections, we will discuss what can limit the wave growth. While at certain conditions the wind can enhance the modulational-instability growth (Waseda and Tulin, 1999), overall effect of the wind on waves appears to be that of a limiter.

Ocean waves of interest to us are always subject to wind forcing. Strong forcing can blow the wave crests away and reduce the limiting wave-breaking steepness and thus reduce the maximal possible surface elevation or wave height (e.g. Babanin et al., 2010). Even weak forcing, however, can change the rate of instability growth, and the parameter range for the instability existence, such as the natural selection of the modulational frequency (e.g. Waseda and Tulin, 1999). Although we are not discussing currents in this paper, it should be mentioned that shear currents and surface currents with velocity gradients can also impose forcing on the surface waves.

Trulsen and Dysthe (1992), based on numerical simulations, showed that modulational instability is delayed in presence of the wind, and delay is greater for stronger forcing. Onorato and Proment (2012), by means of mapping of forced solutions (in both growing and damping conditions) of the Nonlinear Schrodinger Equation concluded that “an initially stable (unstable) wave packet could be destabilized (stabilized) by the wind (dissipation)”.

In experimental studies, Babanin et al. (2010) demonstrated that wind forcing affects both frequency of occurrence of wave breaking and its severity, and slows down the instability growth. Galchenko et al. (2012) demonstrates the effect of the wind forcing on the severity of wave breaking caused by the modulational instability, observed in a laboratory experiments. The effect was quite dramatic.

At extreme wind-forcing conditions, Bliven et al. (1986) in an experiment and Trulsen and Dysthe (1992) in a theoretical study both made conclusions that the modulational instability stops. Babanin (2011b) suggested that wind speeds in excess of U_{10}~33m/s signify a change of the physical regimes in all environments near the air-sea interface: in the atmospheric boundary layer, at the surface and through the upper ocean. Apparently, this cannot be coincidental. We can confidently expect, because of the different physical regime at extreme wind conditions, the probability of occurrence of freak waves to be different to those at lighter wind speeds. Wave evolution is no longer driven by nonlinear interactions at such circumstances, and therefore it would be quite unlikely for the wave statistical properties to be the same as if it was. Most likely, the relative maximal possible wave height should go down in tropical cyclones and extra-tropical storms of similar power.

5. WAVE BREAKING, THE LIMITER

No matter how many waves are superposed and what is strength of the wave instability, the wave heights do not grow unlimited. With massive amount of global wave observations available nowadays, wave heights much in excess of 30m have never been reported (e.g. Babanin et al., 2011b).

The main limiter for wind-generated waves is wave breaking. These waves occupy the wavelength range from meters to hundreds of meters, and if for a given wavelength a wave height grows beyond a certain steepness limit, the water surface becomes unstable and collapses. Thus, for any wavelength there is a natural limit in terms of the wave height.

As mentioned above, the limiting steepness beyond which the breaking will certainly occur is $\varepsilon=ak$~0.45 for progressive waves and $\varepsilon=ak$~0.65 for standing waves. For waves with full spectrum, however, Chalikov and Babanin (2012) showed that, while the limiting steepness can indeed be reached by dominant waves, on average they break at a steepness of $e=ak$~0.2 due to influence of short waves (spectrum tail). We further refer to the book of Babanin (2011a) for an extended discussion of wave breaking problem.

6. CONCLUSION

In the paper, physical reasons behind the generation of rogue waves and the limiters of wave height are discussed. While so far the research community spent most of their effort on understanding the former, here it is argued that understanding the limiters is equally important, in order to be able to predict their occurrence.

In particular, provided the modulational instability in a wave field is active, it appears that it is interaction of the dominant waves with the spectrum tail that is essential. This observations provides an important connection to spectral wave models which are usually employed for the wave forecast. The spectrum tail should be low to let dominant waves grow high because of the modulational instability, before they break, and this can be expected, for example, when the wind drops or turns. For the linear-superposition rogue wave occurrence, conditions of two or more wave systems, with similar peak frequency but different direction, are most suitable.

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