Investigation of Wave Growth and Decay in the SWAN Model: Three Regional-Scale Applications*

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ABSTRACT

Wave growth and decay characteristics in a typical wave action model [Simulating Waves Nearshore (SWAN)] are investigated in this paper. This study is motivated by generally poor agreement between model results and measurements for a regional-scale model of a two-day period during the SandyDuck '97 experiment, wherein there is consistent underprediction of lower-frequency (0.05–0.19 Hz) energy. Two separate methods are presented for improving predictions of low-frequency energy: 1) by altering the weighting of the relative wavenumber term that exists in the whitecapping formulation and 2) by disallowing the breaking of swell. The SandyDuck '97 simulation is repeated with the proposed modifications. Using the first modification, a slight improvement is seen, and with the second modification an apparent problem with nonphysical dissipation of swell by the model is corrected. The modifications are then applied to two other test cases, one in Lake Michigan and the other in the Mississippi Bight. Both cases are of similar scale to the SandyDuck '97 experiment but are freer of uncertainties related to forcing. In both cases, the underprediction of low-frequency energy is observed using the original model, and in both cases agreement with observations is improved via the first of these two proposed modifications. During the course of this investigation, it becomes apparent that, though the model's dissipation term can be improved by these modifications, fundamental problems with the form of the term severely limit the level of improvement that can be achieved.

1. Introduction

The state of the art in medium- and large-scale wave modeling today is the third-generation wave model, which solves the spectral action balance equation without prior assumption of spectral shape (e.g., WAMDI Group 1988; Tolman 1991; Booij et al. 1999). The skill of these models has been demonstrated in numerous validation exercises. Typically, comparisons of bulk parameters such as gross wave height, peak period, and mean wave direction are made, in which these models generally perform well. In deep water, growth and decay are a function of propagation and three primary source terms that are active at all depths: wind input, four-wave interactions, and dissipation. Dissipation in deep water is commonly referred to as ''breaking'' or ''whitecapping.'' Whitecapping is widely believed to be the least accurate of the three terms. That source term is used as a tunable closure mechanism. The models are tuned to match expressions for equilibrium and quasi-equilibrium wave states (typically, Pierson and Moskowitz 1964). Herein lies a problem all of these wave models suffer from, the effects of which may be significant in some situations: the skill of the combined source terms under duration-limited conditions is ignored during the tuning process. The logical result of this is a wave model with questionable performance in duration-limited simulations. One might expect that high-frequency wave components, which generally reach equilibrium state early in the growth process, would tend to be predicted well, while low frequencies may be poorly predicted until they approach equilibrium state. Fetch-limited conditions are often not accounted for in the tuning process either; however, they are modeled with a certain degree of confidence because of the availability of data for validation (e.g., Kahma and Calkoen 1992) and the skill demonstrated in validation against these datasets (e.g., Booij et al. 1999). Such confidence does not exist for the modeling of duration-limited conditions.

We start (in section 2) by describing the model used [Simulating Waves Nearshore (SWAN); Booij et al. 1999] and each of the three deep-water source/sink terms used in the standard implementation of the model, focusing on the least well understood source/sink term, dissipation. Then, in section 3, we discuss the various approaches that other researchers have taken with this term. In this paper, we present three field applications

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of ''regional/subregional'' scale O(100 000 km^2). The first of these is the one that prompted our analysis of the model: SandyDuck '97 (section 4). The simulation is for the time period of 23 and 24 September 1997. The wind speeds are weak to moderate, with a small swell component from the open ocean. In the comparisons with the remotely sensed data (airborne lidar), a significant overprediction of peak wavenumber is evident [this dataset is described by Hwang et al. (2000); comparisons with the SWAN model are made by Rogers et al. (2000)]. We analyze the test case by comparing time series of frequency spectra. There is a general underprediction of low- and medium-frequency energy in the wind sea portion of the spectrum (0.12–0.19 Hz). A second problem is evident: in the model, the presence of wind sea causes swell (approximately 0.05–0.12 Hz) to dissipate. This occurs because of the dependence of the whitecapping sink term on integrated wave steepness. This effect is illogical and is not observed in the buoy data.

It is credible that both of the problems with the SandyDuck simulation are due to low-frequency energy being overdissipated by the model. In section 5, we propose two general types of modifications of the sink term that would tend to reduce dissipation of lower frequencies: 1) alteration of two of the free parameters of the dissipation term used by the model, that of Komen et al. (1984), and 2) by disallowing the breaking of swell. (The definition of swell that we use is given in section 5.) With the second modification, the problem of swell being dissipated during the wind event is corrected; however, neither modification consistently corrects the general underprediction of wind sea energy at and below the spectral peak.

At this point, we are faced with two questions. First, is the problem exhibited in the SandyDuck simulation a persistent problem with the model or just particular to that simulation? Second, what impact would various implementations of the modifications have on the model under other conditions? To answer these questions, we create a second (Lake Michigan) and third (Gulf of Mexico) simulation set of similar scale. Both have the advantage of well-defined boundary forcing (as compared with the SandyDuck simulation). For the Lake Michigan case, we use a moderate-strength, two-part storm event that occurred there during November 1995. Here, the second modification has no appreciable effect because an insignificant proportion of wave energy in this case is considered ''swell'' by our definition. The alteration of the free parameters of the Komen et al. (1984) formulation improves results significantly. In section 8, we validate the modified model using a fetch-limited event in the Mississippi Bight during October 1999. Results from that analysis are nearly identical to that of the Lake Michigan simulation. Sections 10 and 11 present a discussion and summary.

2. Model description

For this investigation, we use SWAN (Booij et al. 1999), version 40.01. SWAN is a third-generation wave action model designed to overcome traditional difficulties[1] of applying wave action models such as WAM (WAMDI Group 1988; Komen et al. 1994) in coastal regions. It uses typical formulations for wave growth by wind, wave dissipation by whitecapping, and four-wave nonlinear interactions (quadruplets or ''quads''). It also includes physical processes (e.g., bottom friction) that are not pertinent to the cases of the present study.

The governing equation of SWAN and other third-generation wave action models is the action balance equation

$$\frac{\partial N}{\partial t} + \frac{\partial C_{g,x}N}{\partial x} + \frac{\partial C_{g,y}N}{\partial y} + \frac{\partial C_{g,\sigma}N}{\partial \sigma} + \frac{\partial C_{g,\theta}N}{\partial \theta} = \frac{S}{\sigma}, \quad (1)$$

where σ is the relative (intrinsic) frequency (the wave frequency measured from a frame of reference moving with a current, if a current exists); N is wave action density, equal to energy density divided by relative frequency ($N = E/\sigma$); θ is wave direction; C_g is the wave action propagation speed in (x, y, σ, θ) space; and S is the total of source/sink terms expressed as wave energy density. In deep water, the right-hand side of (1) is dominated by three terms, $S \approx S_{\rm in} + S_{\rm nl} + S_{\rm ds}$ (input by wind, four-wave nonlinear interactions, and dissipation, respectively). Source term formulations used in wave models are by no means universal, but the default formulations used in SWAN are a fair representation of the mainstream. A discussion of the three source terms follows.

a. Wind input

Wind input in SWAN is expressed as the sum of linear and exponential wave growth:

$$S_{\rm in}(\sigma, \theta) = A + BE(\sigma, \theta). \quad (2)$$

Exponential wave growth (B) is typically larger than linear wave growth (A) by one or more orders of magnitude. For the term B, a SWAN user has the option of using the formulation of WAM cycle 3, due to Snyder et al. (1981) and Komen et al. (1984, henceforth denoted KHH), or the formulation of WAM cycle 4, attributed

[1] The primary ''traditional difficulty'' of applying such models in nearshore regions is that such applications must be computed at high geographic resolution [e.g., O(100 m)]. If a conditionally stable geographic propagation scheme is employed at high resolution, then a high temporal resolution must be used also, which makes computations very expensive. SWAN solves this problem by using an unconditionally stable geographic propagation scheme. In the PROMISE project, the problem was solved in the WAM model by decoupling the time step for propagation and source term integration (Monbaliu et al. 2000). SWAN and PROMISE WAM also include shallow water physics. See Booij et al. (1999) and Monbaliu et al. (2000) for further description of these traditional difficulties.

to Janssen (1991). The default is the WAM cycle 3 formulation,

$$B = \max\left\{0,\ 0.25\frac{\rho_a}{\rho_w}\left[28\frac{U_*}{c}\cos(\theta_{\mathrm{wave}} - \theta_{\mathrm{wind}}) - 1\right]\right\}\sigma, \tag{3}$$

where ρ_a and ρ_w are the densities of air and water, U_* is the wind friction velocity, c is the wave phase speed, θ_{wind} is the mean wind direction, and θ_{wave} is the mean wave direction.

b. *Four-wave interactions*

Four-wave interactions have the effect of transferring energy from the spectral peak to lower and higher frequencies. The energy transfer to lower frequencies leads to lowering of the spectral peak frequency (sometimes referred to as ''downshifting''), and the transfer to higher frequencies leads to increased dissipation by breaking. In SWAN, the discrete interaction approximation (DIA) of Hasselmann et al. (1985) is used. To some degree, the DIA sacrifices accuracy for the sake of computational expediency (see, e.g., van Vledder et al. 2000). Using the DIA in a wave model tends to result in broader spectra than would result using more rigorous methods. The impact of DIA's simplifications on directional spectra is also a concern. However, it has been shown that the bulk parameter prediction (mean frequency, total energy) with the DIA can be quite accurate (e.g., Janssen et al. 1994). The DIA is used for operational forecasts with most, if not all, versions of WAM (e.g., WAM cycle 4; Komen et al. 1994) and in WAVEWATCH (e.g., Tolman and Chalikov 1996).

c. *Whitecapping*

Whitecapping is probably the least understood deep water source/sink mechanism. This dissipation is not easily measured, so prevailing theories provide only vague guidance and the formulas used in wave models tend to be quite empirical. Donelan and Yuan (1994, denoted as DY hereinafter) provide a concise explanation of the whitecapping term used by SWAN, which is based on Hasselmann (1974), KHH, and WAMDI Group (1988).

Donelan and Yuan give an expression for the dissipation sink term that can be rewritten as

$$S_{\mathrm{ds}}(\delta,\ \theta) = C_{\mathrm{ds}}\left(\frac{s}{s_{\mathrm{PM}}}\right)^m \sigma\left(\frac{\sigma}{\sigma_m}\right)^{2n-1} E(\sigma,\ \theta), \tag{4}$$

where C_{ds} is an empirical coefficient of proportionality, s is the overall wave steepness

$$s = k_m\sqrt{E_{\mathrm{tot}}}, \tag{5}$$

the subscript m denotes mean, k is wavenumber, and subscript PM denotes the fully developed sea state [as defined by Pierson and Moskowitz (1964)] for which s is assumed to be a constant. For arbitrary depths,

$$S_{\mathrm{ds}}(\sigma,\ \theta) = C_{\mathrm{ds}}\left(\frac{s}{s_{\mathrm{PM}}}\right)^m \sigma_m\left(\frac{k}{k_m}\right)^n E(\sigma,\ \theta), \tag{6}$$

where n is a free parameter. Donelan and Yuan base this on the following arguments:

- by analyzing negative work done by the whitecap on the wave, it can be shown that dissipation is linearly dependent on frequency and energy density $\partial E/\partial t \propto -\sigma E(\sigma)$;
- the dissipation is dependent on overall wave steepness relative to an equilibrium steepness (s/s_{PM}) to some undetermined degree: $\partial E/\partial t \propto (s/s_{\mathrm{PM}})^m$; and
- the dissipation of an individual wave is dependent on the frequency of the wave relative to a representative frequency of the spectrum to some undetermined degree: $\partial E/\partial t \propto (\sigma/\sigma_m)^{2n-1}$.

The tuning of the whitecapping source term used by SWAN was performed by KHH, who conducted numerical experiments with different whitecapping term coefficients C_{ds} and $2n$ to close the energy balance in deep water and (at the model's duration-unlimited, fetch-unlimited asymptote[2]) match the bulk parameters of the Pierson–Moskowitz spectrum, which was thought to be representative of a limiting spectrum. They employed a relatively rigorous technique for calculating S_{nl} (that of Hasselmann and Hasselmann 1985). KHH reported best results with the two S_{ds} parameters $C_{\mathrm{ds}} = 3.33 \times 10^{-5}$, $2n = 2$ (their M2 case). The values chosen by KHH were the standard for third-generation wave action models until WAM cycle 4 (Komen et al. 1994).

SWAN uses the following expression (Ris 1997; Booij et al. 1999):

$$S_{\mathrm{ds}}(\sigma,\ \theta) = \Gamma\sigma_m\left(\frac{k}{k_m}\right)E(\sigma,\ \theta). \tag{7}$$

Here, the steepness parameter Γ is defined as (e.g., Janssen 1992)

$$\Gamma = C_{\mathrm{ds}}\left[(1 - \delta) + \delta\left(\frac{k}{k_m}\right)\right]\left(\frac{s}{s_{\mathrm{PM}}}\right)^m; \tag{8}$$

C_{ds} and δ are tunable coefficients and s is the overall wave steepness.

SWAN with WAM cycle 3 (WAMDI Group 1988) formulation has $m = 4$, $C_{\mathrm{ds}} = 2.36 \times 10^{-5}$, and $\delta = 0$ (Ris 1997). This is the default formulation in SWAN and the formula used in our baseline simulations. This is equivalent to (6), with $n = 1$, as suggested by KHH.

[2] KHH required a priori that the model reach a quasi-equilibrium state at a particular fetch (i.e., such that increasing the fetch further does not result in increased energy). Since the models with larger values of n generally require a greater fetch to asymptote, they were eliminated.

The value for C_{ds} is equivalent to that given in WAMDI Group (1988), modified because different definitions for mean wavenumber and mean frequency are used. Thus, SWAN's default deep-water source term formulation (represented as the sum of three individual source terms) is tuned to match bulk quantities (total energy and peak frequency) of the (fully developed) Pierson–Moskowitz spectrum via the parameters C_{ds} and n, using $m = 4$.

3. Dissipation in the literature

Hasselmann (1974) theorizes that, since the wave scales are large when compared with the whitecap dimension, the dissipation coefficient should be proportional to the square of the frequency (i.e., $n = 1$). Janssen et al. (1989), using the WAM model with the whitecap model of (6), report too much energy in the higher frequencies using $n = 1$ and more satisfactory results using $n = 2$. They argue that the assumption that wave scales are large in comparison with the whitecap scale is not necessarily valid, especially in the higher-frequency range. Based on this argument, one might envision a formulation where n is dependent on frequency or wave age (equal to unity at lower frequencies, diverging at the higher frequencies). More recently, Janssen (1992) reported optimal results in the high-frequency portion of the spectrum using $\delta = 0.5$ in (8). Their work was incorporated into WAM cycle 4 (Komen et al. 1994). SWAN with the WAM cycle 4 physics has $m = 4$, $C_{ds} = 4.1 \times 10^{-5}$, and $\delta = 0.5$ (Ris 1997). It is worth noting that using $\delta = 0.5$ is essentially a compromise between $n = 1$ and $n = 2$, so it might be expected to yield results similar to (6) with, to give an arbitrary example, $n = 1.5$. [The WAM cycle 4 formulation is not included in this study because it is not the default (recommended) formulation of SWAN. Booij et al. (1999) note that SWAN using WAM cycle 4 formulations gives unsatisfactory deep-water fetch-limited growth. This problem may be due to an unresolved, problematic dependence of third-generation wave models on the high-frequency limit of computations: SWAN uses a (user defined) fixed cutoff frequency, whereas WAM uses a dynamic cutoff. Alternatively, the apparent incompatibility could be due to differences in the numerical implementation of source terms.]

Van Vledder (1999) experiments with tuning the free parameters of the dissipation term of (4) using the SWAN model, along with two DIA parameters (five tuning parameters total). He tuned to the Kahma and Calkoen (1992) growth curve using a relatively short (25 km) fetch case. His tuned source term has only weak dependence of dissipation on the (k/k_m) term. Van Vledder demonstrates a shortcoming with the whitecapping formulation of SWAN [also WAM and early versions of WAVEWATCH (e.g., Tolman 1991)]: an artificial impact of swell energy on wind wave growth. This erroneous behavior of the model is due to the dissipation term being strongly weighted by the spectral mean frequency (or wavenumber), which is in turn very sensitive to the presence of swell. Solutions to this particular problem are under development (e.g., Holthuijsen and Booij 2000).

Banner and Young (1994) investigate the sensitivity of model results to the dissipation source term. Like KHH, their model uses a rigorous formulation for nonlinear interactions (they use Resio and Perrie 1991) and wind input, thus allowing special attention to the impact of the dissipation term on directional spectra. In contrast to KHH, Banner and Young extended computations well into the high frequencies (using an ''unconstrained tail''). They conduct a detailed analysis of fetch-limited growth. They find that no variation of the KHH formulation produced acceptable detailed spectra (as compared with observational data), the shape of the high-frequency tail (important to wave growth characteristics), and in particular, the directional spreading. Their constrained and unconstrained computations result in markedly different high-frequency tail shape. Banner and Young find that their unconstrained tail computations tend to produce too much energy in the tail and that this cannot be corrected (in the context of the KHH dissipation term) without leading to other problems. This, combined with the sensitivity of overall wave growth to the high-frequency tail, indicate that the wave growth of third-generation models is unduly influenced by an artificial feature originally introduced to hasten computations.

Tolman and Chalikov (1996, hereinafter referred to as TC) present new formulations for both the wind input source term and the dissipation sink term. For the dissipation source term, they take the novel approach of distinguishing between high-frequency dissipation and low-frequency dissipation. This is sensible, because the actual physical mechanism for low-frequency dissipation is undoubtedly very different from that of high-frequency dissipation. In their model they use ''low-frequency dissipation'' for all frequencies below $1.75 f_{pi}$, and ''high-frequency dissipation'' for frequencies above $3.0 f_{pi}$, with transition in between (here ''f_{pi}'' refers to the peak frequency of the wind source term, defined in a way that avoids complications by multicomponent sea states). So, in effect, low-frequency dissipation acts on a very large majority of the wave energy, while high-frequency dissipation is limited to the tail region. The latter dissipation is purely diagnostic, an approximate balance on the other two deep-water source terms (wind and four-wave interactions) at higher frequencies. The low-frequency dissipation term is formulated as an eddy-viscosity-type sink term formulation. Tolman and Chalikov start with the expression of Kitaigorodskii and Miropolskii (1968), for total kinetic energy dissipation derived from the Navier–Stokes equations. Using this equation, TC produce a spectral equivalent:

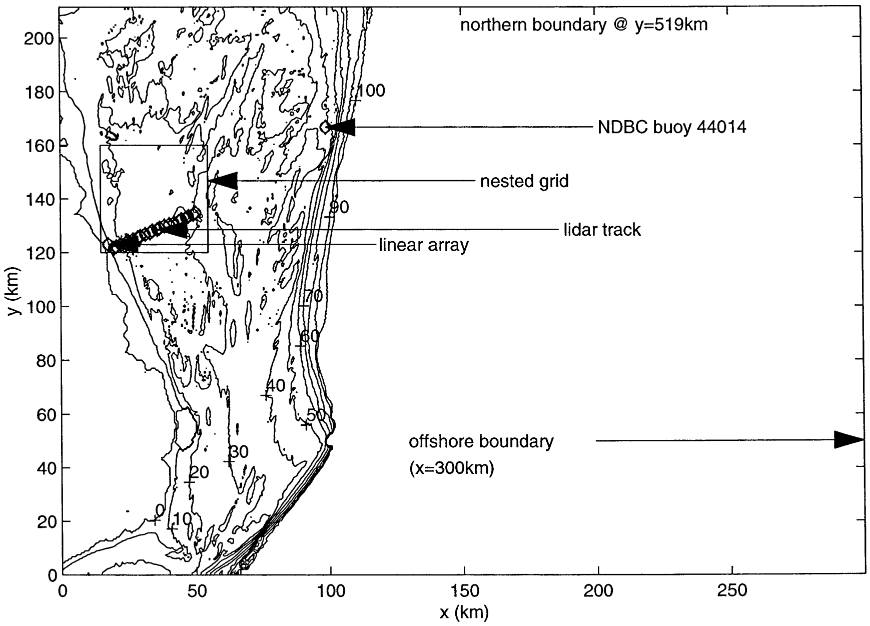

FIG. 1. SandyDuck '97 regional bathymetry with instrument locations and computation grids indicated.

$$S_{\mathrm{ds},l}(f,\ \theta) = 2k^3E(f,\ \theta)\int_0^\infty K(z)e^{-2kz}\,dz, \qquad (9)$$

where K is an eddy viscosity term and z is the vertical coordinate. After dimensional analysis of relevant parameters, they have

$$S_{\mathrm{ds},l}(f,\ \theta) = -2U_*hk^2\phi E(f,\ \theta). \qquad (10)$$

Here U_* is friction velocity, h is the level of high-frequency energy (specifically, the wave height calculated from the high-frequency portion of the wave spectrum, with the lower-frequency bound of the integration being a frequency ''significantly higher than the peak frequency of the wind-sea part of the spectrum''), and ϕ is a dimensionless term related to wave age (specifically, a tuned function of the nondimensional peak wave frequency corresponding to the wind input term).

Other dissipation strategies exist—for example, the quasi-saturated model of Phillips (1985) and Donelan and Pierson (1987) (see also DY), the probability model of Yuan et al. (1986) (see also DY), and Alves and Banner (2000). We omit discussions of these models for sake of brevity.

4. Motivation: SandyDuck '97

a. Description

The SandyDuck experiment, conducted from 22 September to 31 October 1997, provides a wealth of data that can be used for analyzing wave model performance. Wave data sources include two National Data Buoy Center (NDBC) buoys (44014 at the 47-m depth contour and buoy 44004 off of the continental shelf), the Field Research Facility ''linear array'' of pressure gauges (depth 8 m), a set of remotely sensed data [airborne scanning lidar, Hwang et al. (2000)], and other instruments deployed by various researchers. The time period of interest includes a period of active wind wave growth followed by a period of decay.

The simulations are run on two grids: a 2-km resolution grid, extending well past the shelf break, and a nested grid, including the Duck region and extending to approximately 30-m water depth, with 125 m (east–west) × 150 m (north–south) resolution. Regional bathymetry and computational grid domains are shown in Fig. 1. In the present study, comparisons are presented only at NDBC buoy 44014; more detailed model–data comparisons are made by Rogers et al. (2000). For the reader's reference, we provide a table listing relevant model options and controls for all three hindcasts in the appendix.

During this time period, there is a weak swell from the open ocean. Since this swell is relatively steady, we use the ''mean swell condition'' observed during the simulation period for boundary forcing with the larger grid. By using a stationary swell condition, it is possible to isolate (in time series plots) the effect of the model's source/sink terms on swell energy. For wind forcing, we use measurements from three NDBC buoys in the region: 44014, 44004, and 44009, with linear interpolation between the measurement locations. Wind measurements at these locations are shown in Fig. 2. Winds are weak to moderate, with considerable spatial variation; during the wind event on 24 September, winds are predominantly from the northeast.

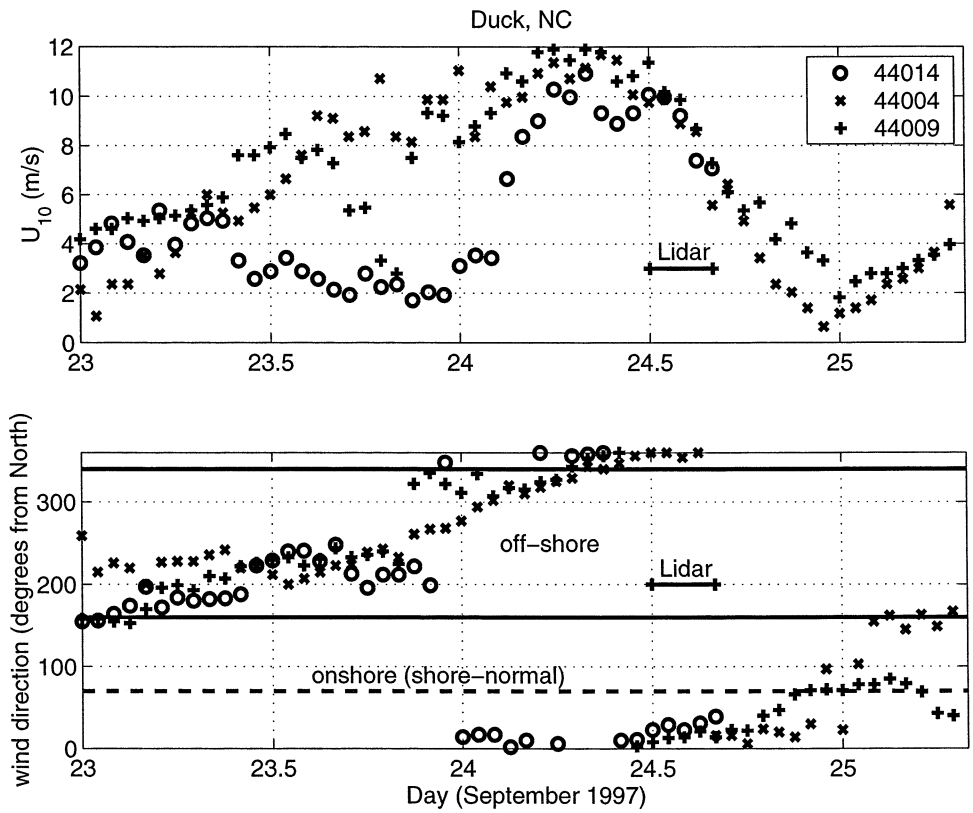

FIG. 2. SandyDuck '97 wind conditions at NDBC buoys in the region. Wind speed has been converted from 5- to 10-m altitude using a power-law relation. Direction is measured clockwise from north (NDBC convention).

b. Results

Comparisons with the lidar data by Rogers et al. (2000) indicate that, while total energy is generally well predicted, mean wave period is significantly underpredicted by the model. In order to investigate this simulation error further, we compare time series of energy at several frequencies (buoy vs model). These are shown in Fig. 3. Here, model spectra are interpolated to the buoy frequencies. Two discrepancies are immediately obvious: 1) energy at and below the spectral peak (approximately 0.18 Hz during the wind event) is underpredicted during the wind event, while energy above the spectral peak is slightly overpredicted, and 2) during the wind event, there is a sudden drop in the swell energy that is not observed in the data.

The latter discrepancy is easily explained: because of the dependence of the dissipation term on the integrated wave steepness, the dissipation of the swell frequencies, previously insignificant, becomes significant in the presence of wind sea. The cause of the first problem is less clear; most likely it is a result of combined inaccuracies of the three deep-water source/sink terms and wind forcing. (Wind measurements during this period of SandyDuck indicate that conditions are quite complex. Thus, there is a higher than normal level of uncertainty with regard to the wind forcing.) To resolve the first problem, we focus our attention on the dissipation term. This is justified by two facts: (i) the dissipation term is the least accurate of the three terms and (ii) the dissipation term is, by design, a closure term and is therefore a means to compensate for inaccuracies in the other two terms until more accurate (or, in the case of S_{nl} computationally expedient) formulations for those two terms are developed.

5. Description of modifications

Below, we detail two modifications that are intended to reduce the two types of problems mentioned above. We have two specific objectives in designing these modifications: 1) that the details of their design should be independent of the motivating simulation (SandyDuck) and 2) that consideration is given to conditions of general nature.

a. Weighting the relative wavenumber

The proper weighting of the (k/k_m) term is uncertain. Increasing the parameter n in (6) has the effect of reducing dissipation on lower frequencies while increas-

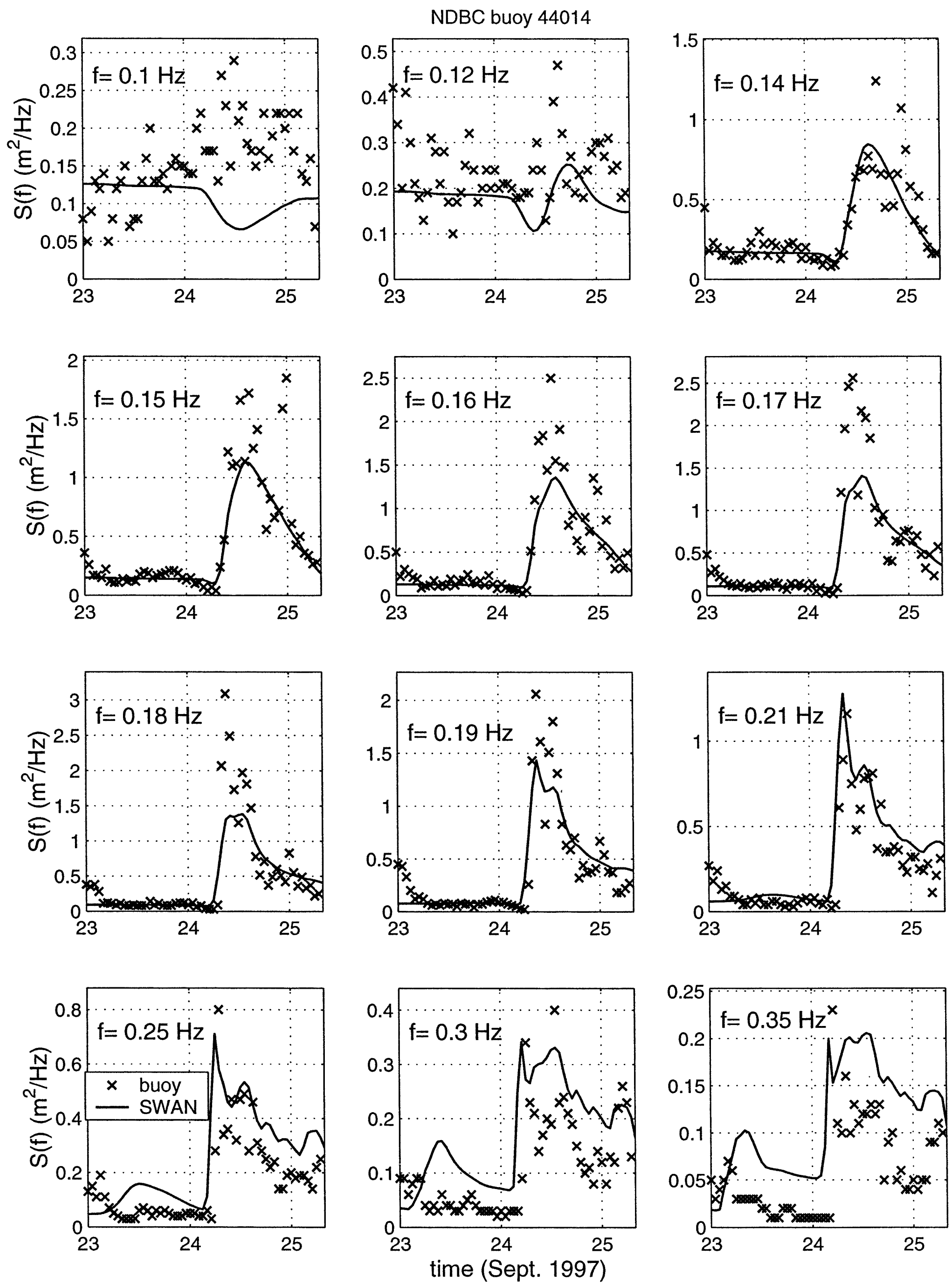

FIG. 3. SandyDuck '97 time series of frequency spectra at NDBC buoy 44014. Buoy data are compared with the SWAN model. Note that scaling of ordinate axis is not constant, so the comparison is essentially normalized.

ing the dissipation of higher frequencies. The value of n used by SWAN ($n = 1$) is based on (i) arguments by Hasselmann (1974), which are unlikely to be valid for the entire wave spectrum, and (ii) the tuning that KHH performed to match the quasi–fully developed asymptote of their model to the Pierson–Moskowitz spectrum. The latter is a concern because one might expect that, because of the use of a different nonlinear solver and less than obvious numerical and implementation issues, the parameters chosen by KHH may not be appropriate for SWAN. For these reasons, a revisit of the KHH investigation for SWAN would seem worthwhile.

If it is desirable for a model to match the Pierson and Moskowitz (1964) limiting spectrum at the model's fetch-unlimited and duration-unlimited asymptote, it is necessary to increase C_{ds} along with n. For a model that uses $n = 1.5$, we determine a value of $C_{ds} = 4.5 \times 10^{-5}$ yields total energy levels similar to that from the original model, for duration-unlimited and fetch-unlimited conditions. During the process of determining this value, it becomes clear that in order to get perfect agreement with the Pierson–Moskowitz values for both the zero-moment wave height (H_{m0}) and the peak wave period (T_p,), a value for n somewhere between 1.0 and 1.5 must be used (we did not attempt to determine the actual value since, given the uncertainties, such precision is not justified). Results from the SWAN model, with both n values, under idealized conditions are presented in section 9. There are a number of ambiguities and contradictions associated with any tuning to the Pierson–Moskowitz spectrum. In fact, it is probably justifiable to disregard the ''equilibrium spectrum'' concept altogether during model design; we discuss this in detail in section 10.

b. Swell dissipation

The steepness-related dissipation term, as used in models such as SWAN and WAM, is applied over the entire spectrum. This seems contrary to the fact that, under all but the more extraordinary conditions, swell in deep water will not break (here we define swell as wave energy, of gentle slope and low frequency, which is not being actively generated by local wind). It can be argued that low-frequency energy might be dissipated by the breaking of higher-frequency waves—through turbulence (in the case of wind sea) or via the ''parasitic capillary effect'' (in the case of much shorter waves). Also, molecular viscosity might be expected to slowly dissipate swell as it propagates extremely long distances. Third, it is conceivable that momentum might be transferred from the waves to the atmosphere. However, these mechanisms of dissipation are completely different from the dissipation of wind sea by whitecapping. Thus, if such dissipation were to be included in the model, it would need to be as a separate process, in a manner similar to that used by TC.

This leaves the question of how to remove swell from the model's breaking process. We pursued the possibilities of a criterion based on frequency, local wave steepness, or wave age, but each of these criteria alone had significant problems in early implementations. Thus we adopted the more conservative approach of requiring that *all three* criteria be satisfied in order for a particular wave component to be considered ''swell.'' The model's dissipation term is modified by

$$S_{\mathrm{ds,modified}}(\sigma,\ \theta) = S_{\mathrm{ds,original}}(\sigma,\ \theta) C_{\sigma,\psi,\zeta}(\sigma,\ \theta), \quad (11)$$

where $C_{\sigma,\psi,\zeta}(\sigma,\ \theta)$ is a combined coefficient calculated from three individual coefficients by

$$C_{\sigma,\psi,\zeta}(\sigma,\ \theta) = \max[C_\sigma(\sigma),\ C_\psi(\sigma,\ \theta),\ C_\zeta(\sigma)], \quad (12)$$

where σ, ζ, and ψ are model variables (ζ and ψ are defined below) and indicate the type of criterion.

A coefficient is equal to zero if a criterion considers the wave component swell and unity otherwise. To prevent potential discontinuities in frequency spectra resulting from any of the three swell criteria, some type of grading of each criterion is necessary:

$$C_\sigma = \begin{cases} 0, & \sigma \le \sigma_1 \\ (\sigma - \sigma_1)\left(\dfrac{1}{\sigma_2 - \sigma_1}\right), & \sigma_1 < \sigma < \sigma_2 \\ 1, & \sigma \ge \sigma_2, \end{cases} \quad (13)$$

and similarly for C_ζ and C_ψ. Here σ_1, σ_2, ζ_1, ζ_2, ψ_1, and ψ_2 are threshold constants (the values used in this study are given below). In (13), we use a simple linear interpolation for intermediate values of σ, ζ and ψ; since we are working with the gradient of the modeled quantity ($\partial E/\partial t$), a smoother function is unnecessary.

The first criterion is based on wave frequency σ, originally motivated by a concern that short waves should always be subject to significant energy losses, regardless of sea state or wind conditions (in the context of the model, if not in nature: short waves in the model that are not dissipated tend to ''linger'' unrealistically even when no longer receiving new energy from the wind, affecting the integrated parameters and thereby retarding new growth.). This criterion can be thought of as a safety mechanism that will ensure that short waves will never be considered swell, in the event that the other two criteria do consider a particular short-wave component swell. [As it turns out, this frequency-based criterion is redundant in our applications and can be omitted without effect (see section 10 below)].

For the second parameter, we define the SWAN model's inverse wave age according to the wind input term of the model:

$$\psi(\sigma,\ \theta) = 28\frac{U_*}{c(\sigma)}\cos(\theta_{\mathrm{wave}} - \theta_{\mathrm{wind}}). \quad (14)$$

The third is a local (in frequency or wavenumber space), steepness-related parameter defined using a small, constant bandwidth Δk over which the spectral density is assumed to be representative[3]:

$$\zeta = k\sqrt{E(k)\Delta k}, \quad (15)$$

where $E(k)$ is related to $E(\sigma)$ via the Jacobian

$$E(k) = C_g E(\sigma). \quad (16)$$

The omnidirectional energy density is used (rather than directional energy density) so that the dissipation modification will not lead to a broadening of the directional distribution of swell components [if directional energy density were used, the peaks (in directional space) would tend to be dissipated, while the other portions of

[3] The term Δk is an artificial construct that exists for the purpose of nondimensionalization. It can be omitted without affecting computations if the ζ term in (16) are correspondingly modified. Note that the frequency/wavenumber discretization of the model cannot be used for Δk, because it typically varies from one simulation to another.

the spectrum would not be dissipated, resulting in a broadening].

For the threshold constants, we use

$$\sigma_1 = 2\pi/(7\ \mathrm{s}) \qquad \sigma_2 = 2\pi/(5\ \mathrm{s})$$
$$\psi_1 = 0.5 \qquad \psi_2 = 1.0 \tag{17a}$$

and based on consideration of fetch- and duration-unlimited (Pierson–Moskowitz) conditions, we choose

$$\zeta_1 = 1.6 \times 10^{-3} \qquad \zeta_2 = 3.2 \times 10^{-3}$$
$$\Delta k = 0.001\ \mathrm{rad\ m^{-1}}. \tag{17b}$$

This choice of steepness-related parameters would correspond to relatively low amplitude swell in a phase-resolved system. Thus, by this definition, only rather mature swells are considered ''swell.'' Young, steep swells will generally not be considered ''swell'' by this definition.

Using this criterion, the impact of the modification on long-duration, large-fetch growth is small. Thus the modification with these parameters can be considered conservative (''conservative modification'' in this context being a modification less likely to produce negative side effects, such as excessive growth or oddly shaped spectra). For example, in simulations of wave growth from rest with infinite fetch, the total wave energy after 10 days shows only small to moderate increase in comparison with the unmodified model. In fact, for 10-m wind speeds less than approximately 15 m s^{-1}, this ''long fetch, duration'' energy level in the modified model is still below that given by the Pierson–Moskowitz spectrum. (The wind speed is important here, since the P–M spectrum scales with the 10-m wind speed and the model does not.)

6. SandyDuck '97: Revisited

The SandyDuck '97 simulation (section 4) is repeated with the following modifications:

- $n = 1.5$, $C_{ds} = 4.5 \times 10^{-5}$ [C_{ds} is set to a value that will yield total energy levels similar to the original model ($n = 1.0$, $C_{ds} = 2.36 \times 10^{-5}$), for fetch-unlimited and duration-unlimited conditions], henceforth denoted n1.5PM;
- $n = 2.0$, $C_{ds} = 2.36 \times 10^{-5}$ (included for comparative purposes, this C_{ds} is the default setting for $n = 1$; thus this setting is not tuned to reach an equilibrium that matches the Pierson–Moskowitz spectrum), henceforth denoted n2.0; and
- $n = 1.0$, $C_{ds} = 2.36 \times 10^{-5}$, with no breaking of swell, as defined by criterion (11) (these values of n and C_{ds} are those of the unmodified S_{ds} term), henceforth denoted n1.0PM,NDS.

These results are compared with the original model (n1.0PM) in Fig. 4. The nonphysical decrease in swell energy is clearly eliminated by modification (n1.0PM,NDS). Since the swell energy is small for this case, the effect of the correction on total energy level is slight. However we expect that, in cases where swell energy is, by proportion, more substantial or in cases where the modeler is primarily interested in low-frequency energy, this correction will provide significantly more accurate results than the original SWAN model. This modification is also relevant to a model such as WAM, though the magnitude of the problem being corrected is considerably smaller in the case of the WAM model (P. Janssen 2001, personal communication). Note that steep swells are not considered ''swell'' using the parameters given in (16). This is discussed further in section 10.

Modifications (n1.5PM) and (n2.0) have the anticipated effect of increasing low-frequency energy and decreasing high-frequency energy (generally improving results). However, modification (n2.0) causes an overprediction of growth in the range 0.12–0.14 Hz, suggesting that, at least for this case and on the low-frequency face of the wind sea spectrum, digression from the infinite-fetch, infinite-duration asymptote of the original model is not beneficial.

7. Analysis: Lake Michigan

a. Data

A two-part storm event in Lake Michigan during November 1995 is modeled. The lake bathymetry, provided by the NOAA/Great Lakes Environmental Research Laboratory, is shown in Fig. 5. For the time period of interest, data are available at four locations: 1) NDBC buoy 45002, located in the northern part of the lake in relatively deep water (>100 m); 2) C-MAN station SGNW3, located at a central latitude at the western shore; 3) NDBC buoy 45010, near the western shore in relatively shallow water (~15 m); and 4) NDBC buoy 45007, the southern deep-water buoy (depth > 100 m). The buoys provide wind and wave data, while the C-MAN station provides only wind data. Wind conditions at all four data locations are shown in Fig. 6. An approximately 30-h wind event occurs, with winds generally from the south, peaking at approximately U_{10} = 17 m s^{-1}. After a brief lull, there is a slightly stronger event, with duration approximately 36 h, winds predominantly from the north, and U_{10} peaking around 20 m s^{-1}. Apart from a mild sheltering effect evident in the nearshore measurements, the winds seem to be relatively homogeneous over the lake. Air–sea temperature differences measured by the buoys are small.

Both events are characterized by a growth phase (during which wind speed and wave energy grow rapidly), a brief transition phase (when wind speed is highest and source, sink, and propagation terms roughly balance), and a decay phase (when wind speed is decreasing, waves decay via whitecapping and propagation). The decay of the second event is caused by a simultaneous decrease in wind speed and fetch length. Note that for buoy 45007, the second, northerly event is of signifi-

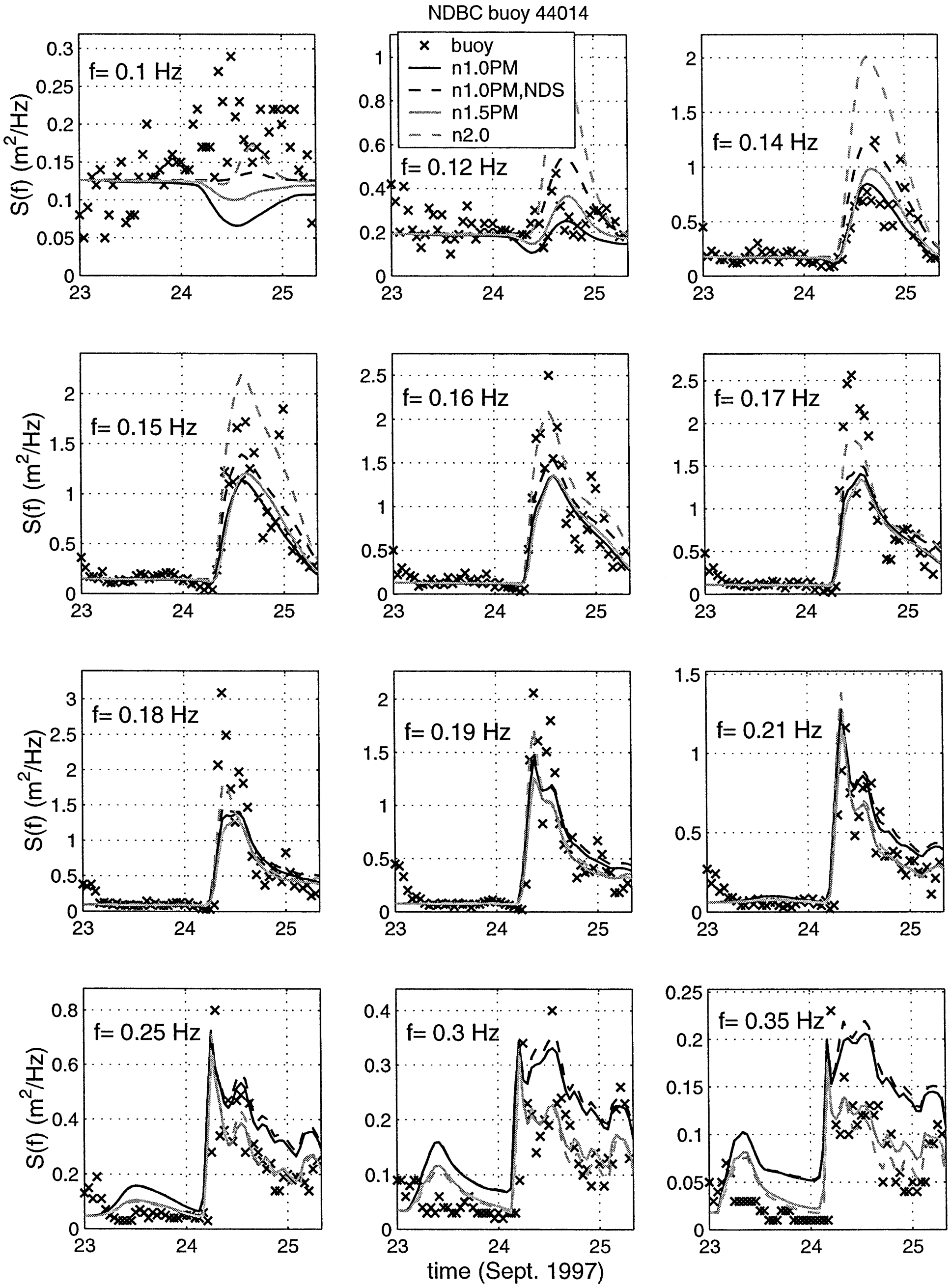

FIG. 4. As in Fig. 3 but the SWAN models with modified S_{ds} formulations are also shown.

cantly longer fetch than the first, southerly event, while for buoy 45002, the reverse is true. Thus, considering the buoys separately, we are modeling two short fetch events and two longer fetch events with this simulation. Bulk wave parameters measured by the two buoys are summarized in Table 1.

b. Model setup

The two events are modeled in one simulation, initialized with zero energy state at 1600 local time on 8 November. Wind data from buoys 45002 and 45007 are used. Wind data from the two coastal data locations are not used because of the sheltering apparent in data from those locations (wave comparisons are made at the two open-water locations, where sheltering effects are less important). Wind is simply assumed uniform in *x* (longitude), with variability in *y* (latitude) determined by interpolation between the two data points. Model results are compared at the locations of buoys 45002 and 45007. See the appendix for additional details on model setup.

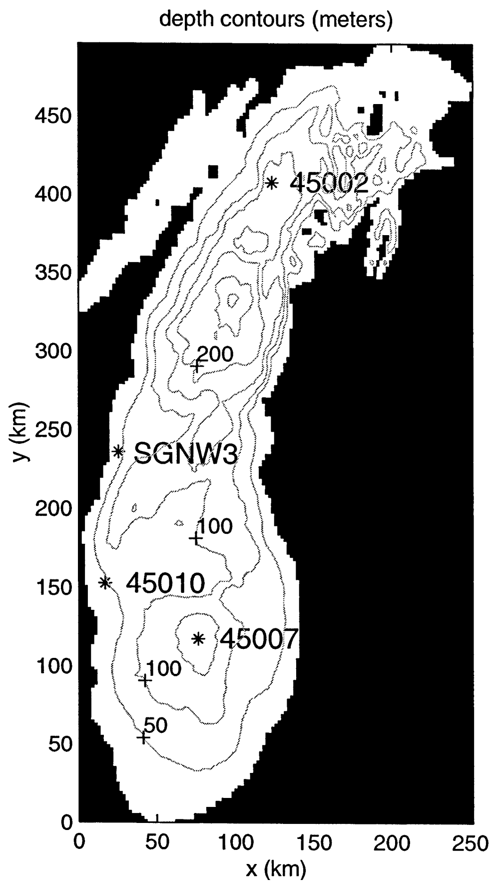

FIG. 5. Lake Michigan bathymetry and instrument locations.

c. Comparisons with data

1) UNMODIFIED MODEL RESULTS

Three of the four models are compared with data in Figs. 7a,b. Since we use wind speeds measured at unsheltered (deep water) locations and applied over the entire domain, one expects that input winds speeds are higher than truth near the shoreline, where the nearby topography tends to reduce wind speeds. This would tend to create an overprediction of wave growth. However, the error of the original model is (for the most part) an underprediction of energy in the range 0.08–0.15 Hz. This points clearly to a deficiency in the source/sink terms (as opposed to forcing). The underprediction of low-frequency energy is most evident in the long-fetch events. This is consistent with the SandyDuck simulation (section 4), suggesting that where the fetch is large (>100 km) (and thus wave growth is more duration-limited than fetch-limited) the model underpredicts low-frequency energy. Figures 7a,b show that high-frequency energy is generally well predicted, with a slight overprediction at the highest frequencies. For the short-fetch events, wave height and mean period (not shown) are generally well predicted. However, we note that these events would be more affected by the neglect of sheltering (by nearby topography) in the wind forcing. Thus it is possible that for the short-fetch events, weak wave growth in the model is being compensated by exaggerated wind speeds near the coastline.

2) MODIFIED MODEL RESULTS

With the n1.5PM model, the impact is seen mostly in the higher frequencies (which are dissipated more because of higher n and higher C_{ds}). In the lower frequencies, the higher n and higher C_{ds} have an opposing effect on dissipation and largely balance each other.

The n2.0 model produces a clear improvement in results, especially at buoy 45007. At buoy 45002, there is a moderate overprediction of some energies for the short-fetch case, but we expect that an exaggeration of wind speeds near the coastline is at least partially to blame for this. Otherwise, the improvement is quite significant.

The n1.0PM model and the n1.0PM,NDS model yield results that are almost identical. This indicates that the swell criterion is working as intended, since swells do not occur in this application. (Because of rapidly shifting winds in this test case, it should be expected that "young swell" does have a strong presence in the simulation. However, steep, young swells are not considered swell in our definition. Steeper swells can be included in the definition by increasing ζ_1, ζ_2.)

8. Verification: Mississippi Bight

a. Data and model setup

We have shown that with an altered dissipation formula, the skill of the model can be improved for the SandyDuck and Lake Michigan test cases. We use a third application—a moderately strong wind event in the Mississippi Bight—as a check on the consistency of the effects that the modifications have on the model.

The SWAN model is being developed as a near-real-time wave model for the Mississippi Bight region as a component of the Northern Gulf of Mexico Littoral Initiative. Figure 8 shows the computational grid and the location of two NDBC buoys. The test case used herein was designed by Hsu et al. (2000) as a preliminary simulation for the nowcast model, covering the time period of 0000 UTC 19 October through 0100 UTC 22 October 1999. Wind data from both buoys are shown in Fig. 9. It is an ideal test case: winds range from weak to moderately high strength and are directed offshore (fetch limited). Very little wave energy enters the computational domain through the open ocean boundary, minimizing concerns of error in specification of the boundary conditions. Wind and wave data are available within the computational grid from two NDBC buoys. The wind field appears to be fairly uniform between the two buoys during this period and is generally from a northerly direction. Therefore, the model is forced with

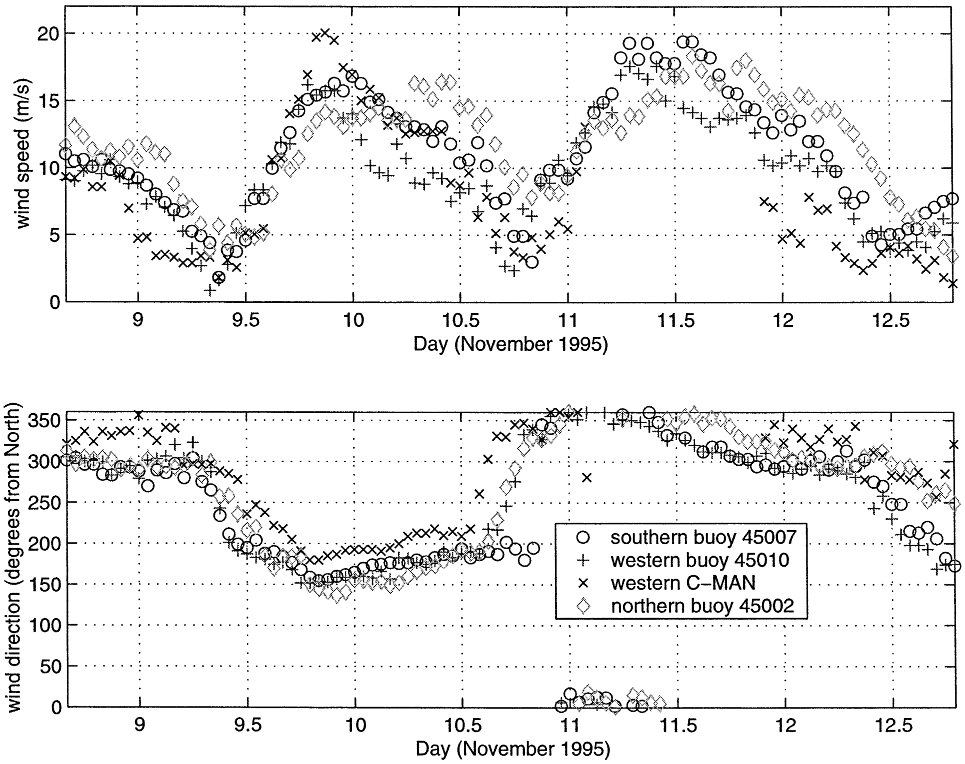

FIG. 6. Lake Michigan wind data at four instrument locations. Wind speed has been converted from 5- to 10-m altitude using a power-law relation. Direction is measured clockwise from north (NDBC notation).

uniform wind input, based on data from buoy 42040. The bottom friction formulation from the Joint North Sea Wave Project (JONSWAP: Hasselmann et al. 1973) is used. (See the appendix for additional details on model setup.)

b. Model–data comparisons

Figure 10 compares the model results with data from buoy 42040. Results are very consistent with the Lake Michigan test case.

- The n1.0PM and n1.0PM,NDS results are almost identical. This is unsurprising, because there should be very little of any kind of swell in this simulation.
- The n1.0PM and n1.5PM results are similar, except in the higher frequencies, where a higher n and higher C_{ds} both contribute to greater dissipation.
- The n2.0 model is significantly more accurate than the other models, with the exception being in the (relatively low energy) 0.12–0.13-Hz range, where the n2.0 model overpredicts energy somewhat. This overprediction is due to the broadness of the model spectrum relative to the buoy spectrum. The spectral width is not particularly sensitive to our variations of the dissipation function. It is almost certainly controlled primarily by the S_{nl} term. So there is probably a limit to how much skill can be gained within the framework of the KHH dissipation functional form without a more rigorous nonlinear solver (e.g., Resio and Perrie 1991). Nonetheless, improvement here with n2.0 is considerable.

TABLE 1. Bulk parameters from NDBC buoy measurements, Lake Michigan, 9–13 Nov 1995. Bulk parameters at the peak of each event are given. Mean period, T_{m01}, is the centroid of the wave spectrum. Wave height and mean period are based on spectrum from 0.07 to 0.4 Hz.

H_{m0} (m)	T_p (s)	T_{m01} (s)
	Buoy 45007	
3.7, 5.2	7.7, 10.0	7.5, 9.2
	Buoy 45002	
4.6, 3.0	10.0, 6.2	8.6, 6.1

9. Verification: Idealized wave growth

As mentioned above, the models n1.0PM, n1.0PM,NDS, and n1.5PM are designed such that with fetch-unlimited and duration-unlimited conditions (for the latter we use a simulation length of 30 days), the result will be close to the Pierson–Moskowitz limit. The model n2.0 was not designed to achieve this end result (for this model, n was modified with no corresponding retuning of C_{ds}). In order to quantify, in some way, the

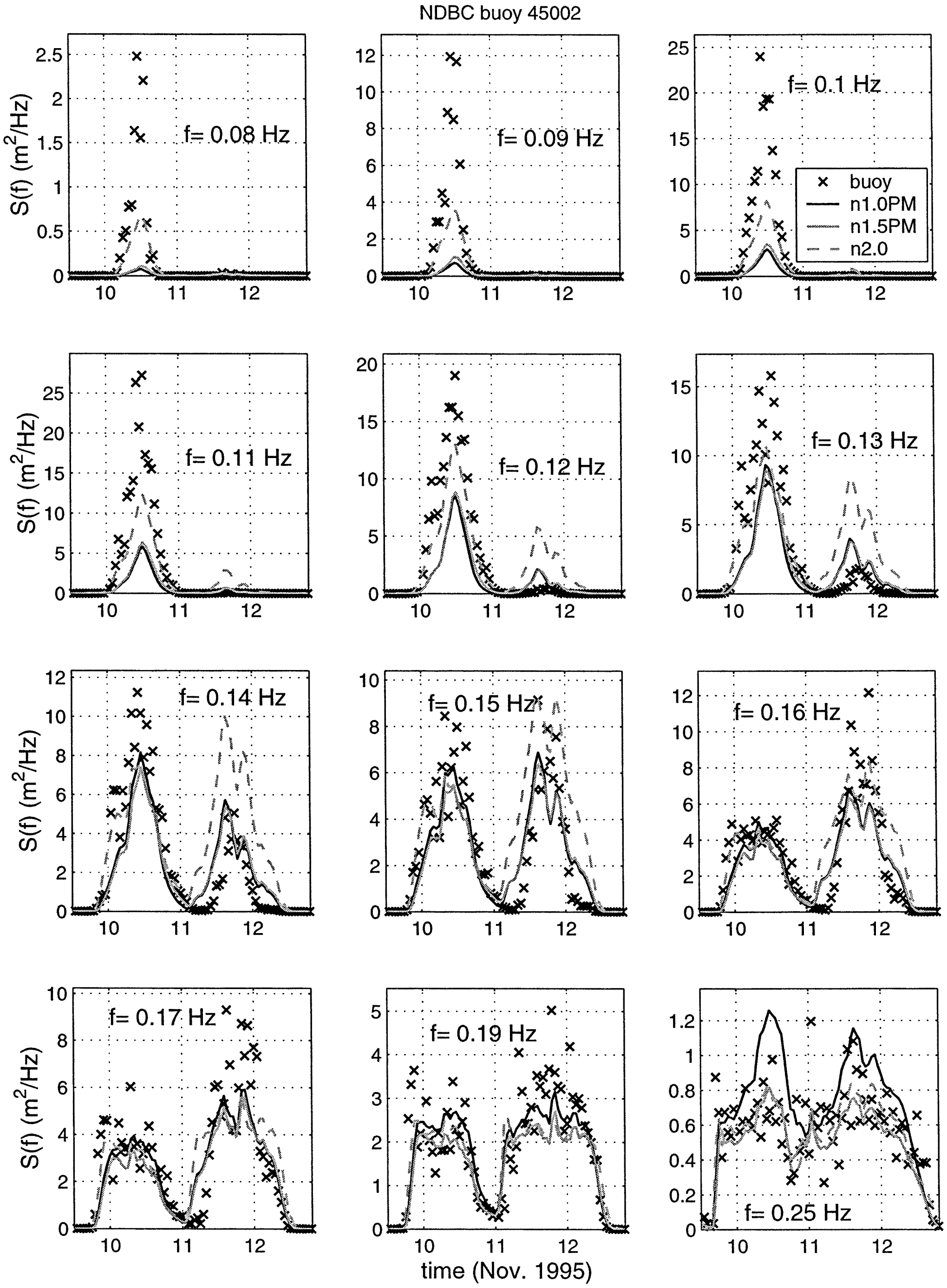

FIG. 7. Lake Michigan time series of frequency spectra at two NDBC buoy locations. The SWAN models are compared with measurements. The n1.0PM,NDS model is not included in this comparison, because it is practically identical to the n1.0PM model in this simulation: (a) NDBC buoy 45002 and (b) NDBC buoy 45007.

rapidity (in both a temporal and spatial sense) in which the models approach (or, in the case of the fourth model, exceed) the generally accepted asymptote, we present results from an idealized duration- and fetch-limited case (U_{10} = 15 m s^{-1}; Fig. 11). With this wind speed the Pierson–Moskowitz wave height is $H_{m0,\mathrm{PM}} = 5.5$ m and the duration- and fetch-*un*limited asymptote of the n1.0PM model (the unmodified SWAN model) and the n1.5PM model are both $H_{m0} = 4.8$ m. We see that the removal of swell from the dissipation process has only a minor impact on the result since most wave components are active wind sea in this case. It is noteworthy that, though the n1.0PM and n1.5PM models have similar asymptotes, the n1.5PM model appears to require slightly more fetch to achieve this (it has a flatter fetch-growth curve).

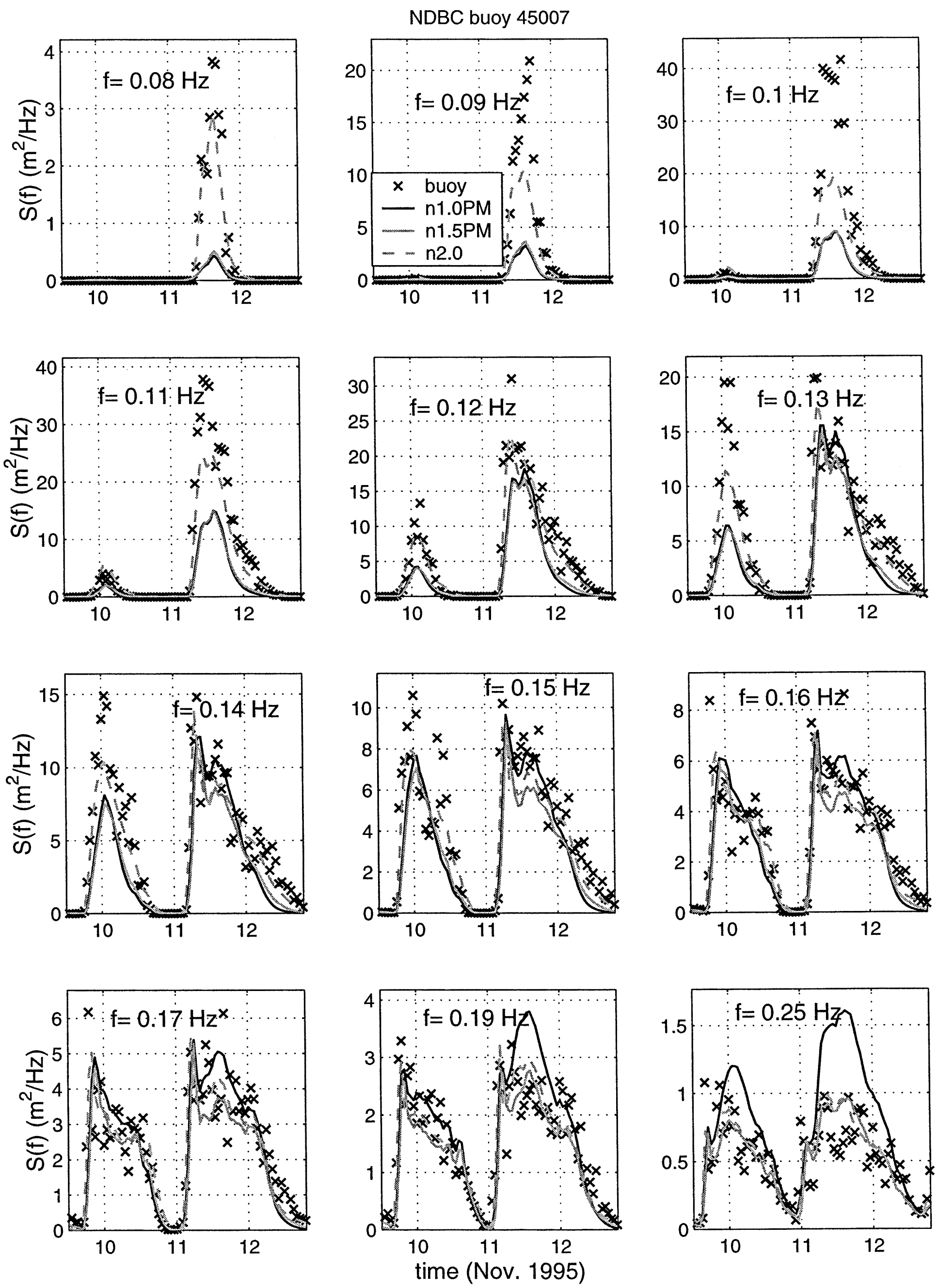

FIG. 7. (*Continued*)

The median fetch and duration of wind events reported by Moskowitz (1964) are 350 km and 12 h. Figure 11 shows that at these fetches/durations, the n2.0 model is much closer to the Pierson–Moskowitz energy level than are the models with lower *n* values. The n1.0PM and n1.5PM models have an asymptote close to the Pierson–Moskowitz energy level but achieve this asymptote too slowly. The n2.0 model exceeds the Pierson–Moskowitz energy level at larger fetch/duration, but this occurs where little or no data exist to confirm or dispute the model result. The tuning to Pierson–Moskowitz limits is discussed further in section 10.

10. Discussion

a. Swell definition

For our definition of ''swell'' (11), (12), we use a combination of three criteria. The definition is such that, for a given wave component, at a given time and lo-

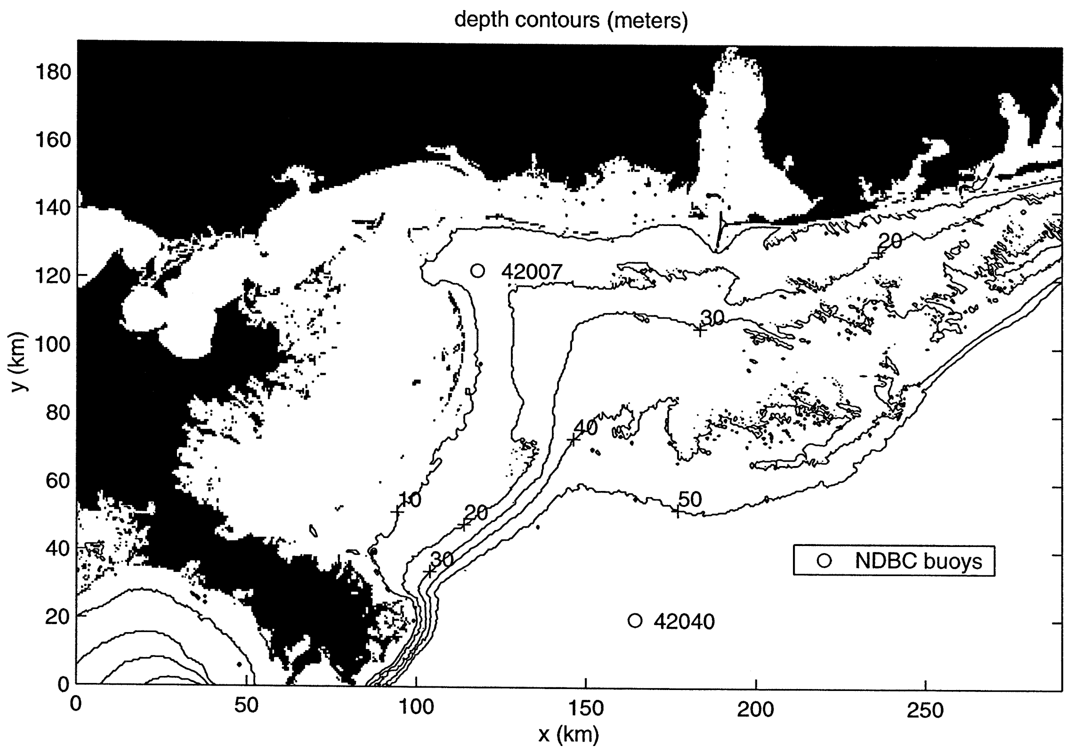

FIG. 8. Mississippi Bight bathymetry and instrument locations.

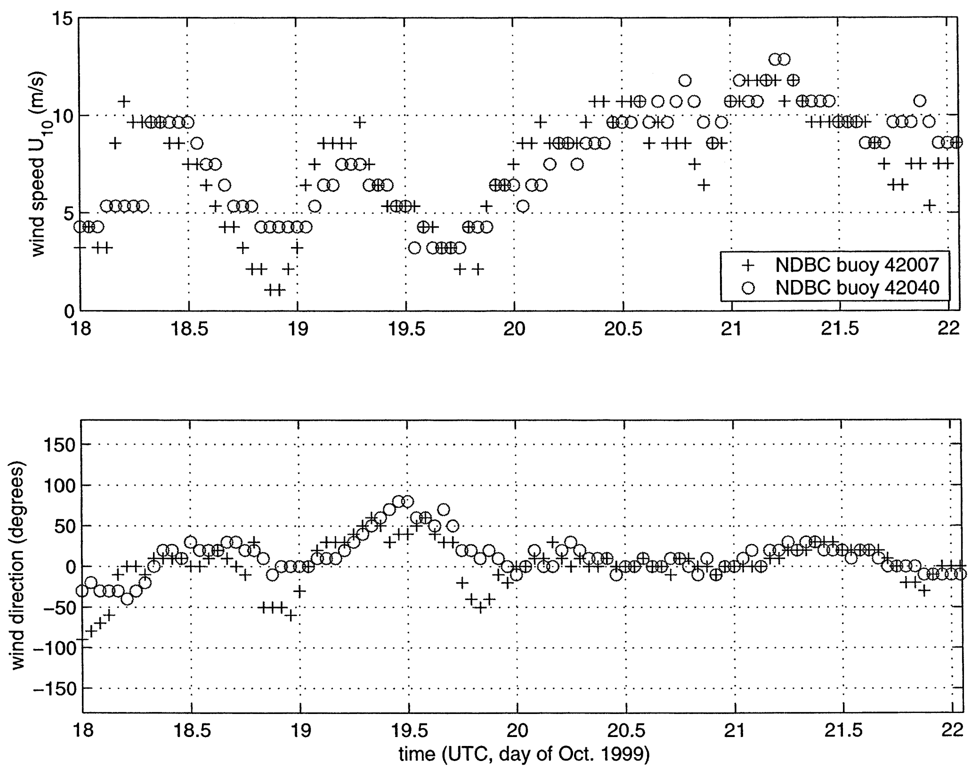

FIG. 9. Mississippi Bight wind data measured at the two NDBC buoys in the region. Wind speed has been converted from 5- to 10-m altitude using a power-law relation. Direction is measured clockwise from north (NDBC notation).

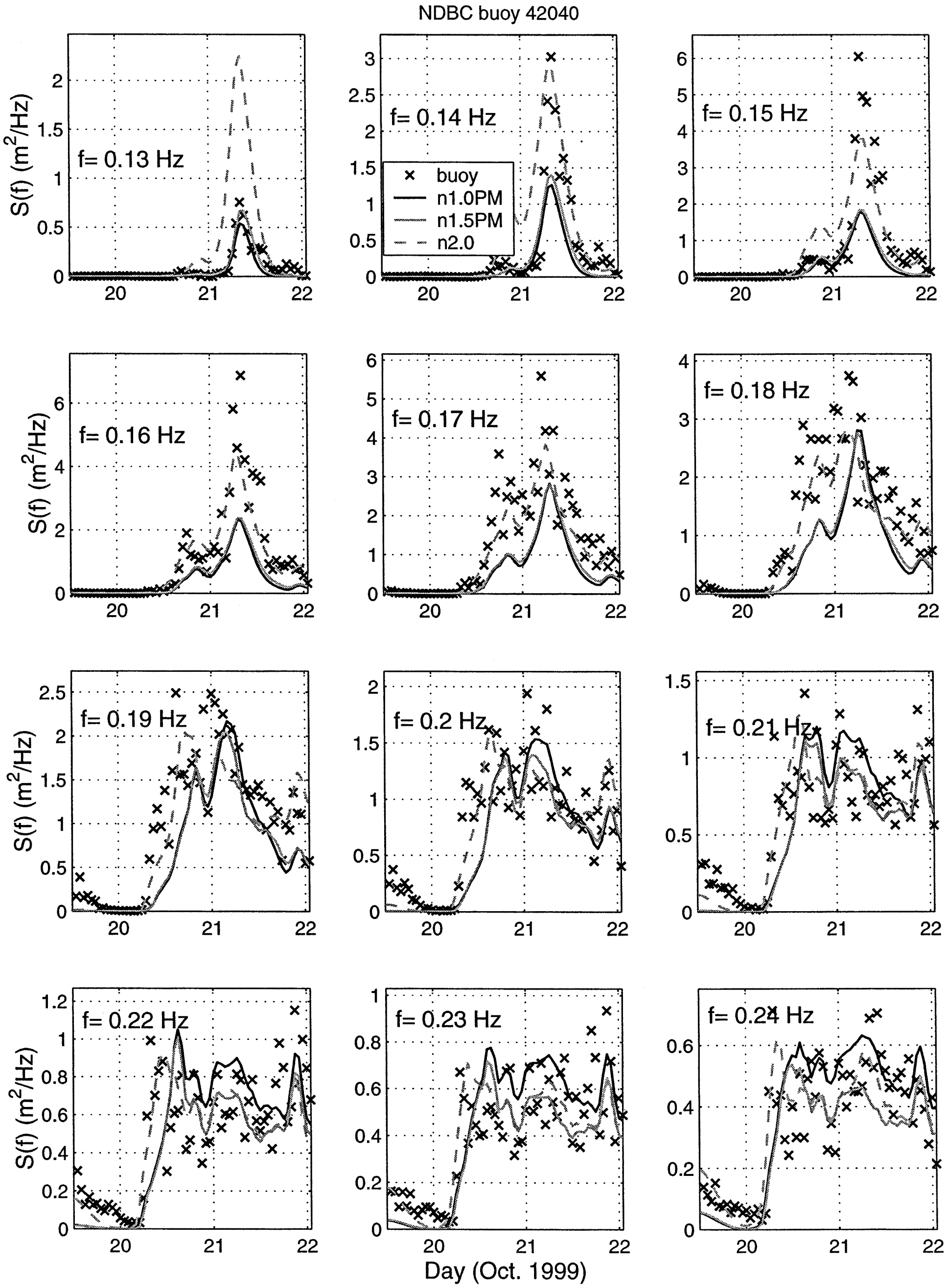

FIG. 10. Mississippi Bight time series of frequency spectra at location of NDBC buoy 42040. The SWAN models are compared with measurements. The n1.0PM,NDS model is not included in this comparison, because it is practically identical to the n1.0PM model in this simulation.

cation, only one criterion will affect the calculation (if any). This naturally leads to the question of whether one or more of the criteria can be omitted, to simplify the modification. For the three field cases presented here and for a canonical infinite-fetch, long-duration case we have investigated this question and made the following observations.

- All models that include the steepness-related criterion yield favorable results. All models that exclude the steepness-related criterion perform poorly in at least one instance. This suggests the possibility that only the steepness-related criterion is needed.
- When the steepness-related criterion is used, including the wave-age-related criterion does improve results

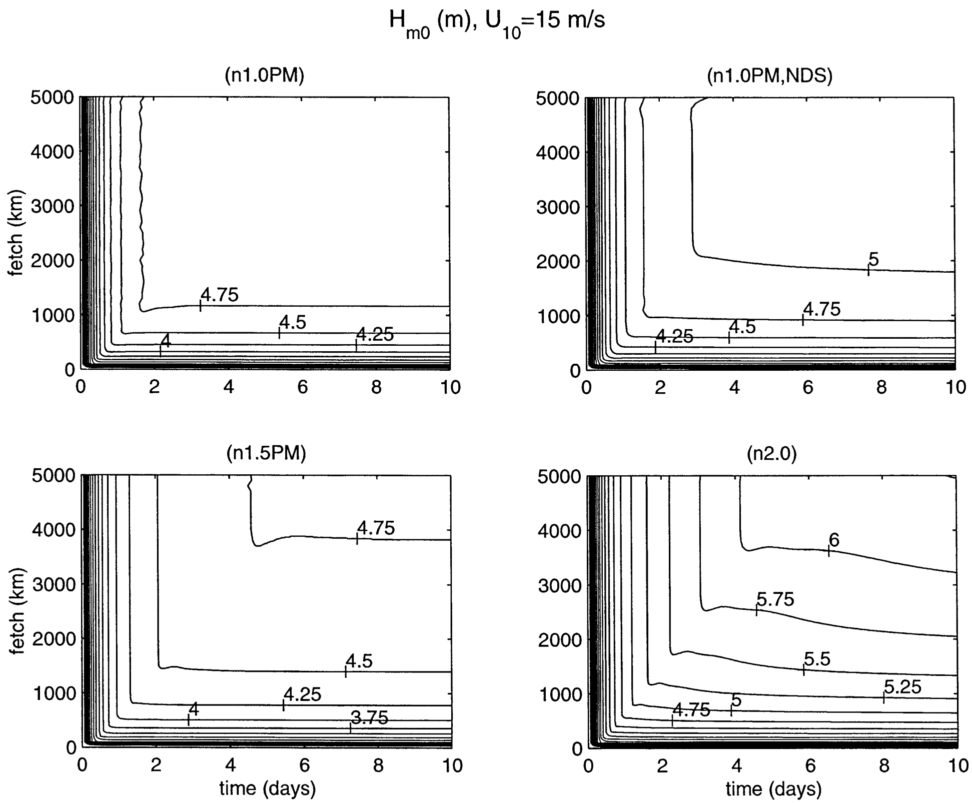

FIG. 11. Fetch-limited growth curves of the four SWAN models. Wave height as a function of fetch and time is shown for the case of infinite depth and U_{10} = 15 m s^{-1}. The Pierson–Moskowitz spectrum predicts a wave height of 5.5 m for this wind speed. The median fetch and durations of wind events reported by Moskowitz (1964) are 350 km and 12 h. (Note, of course, that those measured events do not start in a state of rest, as is the case with the models.)

further, slightly. [In the Lake Michigan and Mississippi Bight cases, we get a result closer to that of the unmodified model (preferred because of the absence of swell in these cases) if we include the wave-age-related criterion in addition to the steepness-related criterion. And in the SandyDuck case, a large portion of the wind sea spectrum is incorrectly considered ''swell'' when the wave-age criterion is omitted.]

- The impact of the frequency-related criterion is insignificant in all cases.
- A model with the frequency-based criterion and wave-age-based criterion (without the steepness-related criterion) performs well in the field applications presented here but overpredicts downshifting of energy in the canonical (long fetch, duration) test case. This overprediction is possibly due to inaccuracies of the DIA routine used for S_{nl}. This is discussed further in section 10e(1).
- A model with only the wave-age-based criterion produces a peculiar result. This model tends to nullify dissipation in more instances than does the model with the combined criterion (presented in this manuscript). However, the total wave energy is reduced in this model in many instances. This appears to be due to the presence of high-frequency energy propagating in a direction for which no energy is received from the wind (i.e., against the wind), which are therefore (erroneously) considered ''swell'' by the model. Overprediction of high-frequency energy leads to an overprediction of dissipation of the entire spectrum through the integrated parameters in the dissipation term.

These conclusions are, of course, influenced by the chosen values of σ_1, σ_2, ζ_1, ζ_2, ψ_1, and ψ_2. For example, larger steepness-related parameters would lessen the impact of the steepness-related criterion, and thus the wave-age criterion would play a larger role in the applications presented.

The primary drawback of this model modification (disallowing the breaking/dissipation of swell) is that in order to make it generally applicable (such that it does not create problems under any circumstances), we must use a rather strict (conservative) definition of swell, thus limiting its usefulness to correction of aphysical dissipation of low-steepness swells (i.e., aphysical dissipation of moderate-steepness swells might not be corrected). We anticipate that with a more accurate representation of the downshifting of energy in long-fetch, long-duration cases (e.g., via a more accurate S_{nl} term), the steepness-related criterion in our definition of swell could be relaxed, thereby increasing its usefulness.

This modification is—superficially, at least—simple and direct, and performs quite well in the four applications presented in this paper. However, the complexities associated with the modification underscore its imperfection. We expect that a ''painless'' solution to the problems associated with the KHH form is not possible. The need for an altogether different dissipation form is apparent.

b. The influence of propagation numerics

It would be misguided to alter the physics of a numerical model to improve predictions without first determining that error is not—either wholly or partially—a result of numerics. Bender (1996) shows that in the Southern Ocean (where swell has a strong presence), the error of a third-generation wave model (WAM cycle 2) with the wind generation mechanism of Snyder et al. (1981) is caused primarily by the first-order numerics of the model. In this case (according to Bender), the model is better served by upgrading the propagation scheme, as opposed to adopting the more recent wind input formulation of Janssen (1989, 1991) that is used in WAM cycle 4.

The version of SWAN used in this study (v40.01) employs a relatively diffusive numerical scheme for geographic propagation, the unconditionally stable first-order, upwind, implicit scheme. A time step size of 6.0 min was used, corresponding to a rather diffusive Courant number of about 1.8 for a fast ($T = 13$ s) wave. However, numerics are not expected to have a strong influence on the Lake Michigan simulation because the curvature of the wave action field in the simulation is generally mild and well resolved by the computational grid. The simulation is repeated with improved propagation numerics and it is determined conclusively that numerical error is minimal. Propagation numerics are improved by using the second-order scheme of Stelling and Leendertse (1992) ($Q_0 = 0$, $Q_1 = 1/6$), and a smaller time step (2.2 min). Note that the negligible effect of the smaller time step also verifies that the spectral propagation is well resolved in time with the coarser (6.0 min) time step.

c. Influence of SWAN's convergence feature

Run in stationary mode, SWAN requires multiple iterations to compute the solution accurately [a result of the model's quadrant-sweeping solution procedure; see Booij et al. (1999)]. Run in nonstationary mode (which is the mode used in the cases presented in this manuscript), the model allows multiple iterations per time step, but the default is to omit iterations since, if the time step is sufficiently small (i.e., if the problem being solved does not differ greatly from one time step to the next), there is no need to iterate at each time step. We omit iterations in our calculations. The experiment with smaller time step sizes (previous section) implies that omitting iterations does not affect our calculations. (We repeated the Mississippi Bight simulation with multiple iterations as an additional confirmation.)

d. S_{in} *and* S_{nl4}

Underprediction of low-frequency energy can be attributed to one or more of the three deep-water source/sink terms. Because the whitecapping term is the least accurate of the three terms, we focus our efforts on analyzing this term and improving model accuracy through modification of this term. However, it should be stressed that the whitecapping term remains a ''closure'' term, the function of which is (in part) to compensate for the inaccuracies of the other two terms, which are by no means small. Therefore, if model skill is improved through modification of the dissipation term, this does not necessarily indicate that the skill of the dissipation term itself has been improved. Further, if a more accurate S_{in} term or an accurate and computationally expedient S_{nl4} term is later available for operational forecast models, the dissipation term will obviously need to be revisited.

e. Tuning to duration-unlimited conditions

1) THEORETICAL CONCERNS

The formulation of deep-water source/sink terms for a third-generation wind wave model invariably includes a tuning process. This is typically done by tuning the dissipation term such that the bulk parameters (e.g., wave height, peak period) match an empirically based ''quasi-equilibrium'' target value at the model's infinite-duration asymptote, or (in the case of WAM cycle 3 and SWAN) infinite-duration *and* fetch asymptote. Such simplification is often desirable, in order to limit the degrees of freedom in the tuning process. In a temporal sense, the asymptotes of these models may be well tuned, while the accuracy of the rate at which the models approach these asymptotes is uncertain. Modification of a model to more accurately simulate duration-limited cases may be viewed as one logical subsequent step in the development process, though of course one expects this to require more complexity in the tuned source term(s).

The question arises of whether it is appropriate to tune a model to quasi-equilibrium (Pierson–Moskowitz) limits at all. Though the concept of a duration-unlimited state conveniently removes time dependency from the tuning process, it also involves a certain disconnect from reality. For example, with $U_{10} = 10$ m s^{-1} and infinite fetch, the original SWAN model reaches an equilibrium state after one day. Though one would obviously expect development to slow dramatically, it is unrealistic to expect that *no downshifting* would occur after one day, given infinite fetch. It is more logical that some energy would continue to be transferred to lower frequencies by nonlinear interactions and that dissipation of these long, low-steepness waves would be minimal. In fact, the model does transfer energy to these lower frequencies continually, but growth there is arrested by nonphysical dissipation of these long, low-steepness waves. It is possible that this nonphysical dissipation is necessitated by an overprediction of energy transfer to the low-frequency face by discrete interaction approximation for S_{nl}. Since duration-unlimited, fetch-unlimited forcing conditions do not occur in nature, these may seem like philosophical issues. However, it is relevant insofar as the tuning process may be poorly conceived.

Observations of a model modification not presented in this paper shed some light on this: a model with only the wave-age-related and frequency-related swell criteria (i.e., excluding the steepness-based criterion) performs well in the three field applications (with nonstationary forcing) but greatly exceeds the Pierson–Moskowitz energy level in the canonical, long-duration, infinite-fetch case. The latter is likely due to this slow, steady ''leakage'' of energy to low-frequency waves that are considered swell because they do not receive energy directly from the wind. Again, this leakage in the model is physically appropriate but may be overpredicted by the DIA.

2) Practical concerns

In our case, tuning a model to asymptotically reach a certain value is disadvantageous. Our primary goal is to correct a consistent underprediction of lower-frequency energy. Yet it is the development of these lowest frequencies that controls asymptotic behavior. Thus, if we dictate that the modified model must reach the same asymptote as the original model, the dissipation of the lower frequencies must be similar. To state this another way, at lower frequencies an increased C_{ds} tends to balance an increased n (the differences between the n1.0PM and n1.5PM models are generally only significant at the higher frequencies). The increase of n without the *corresponding* retuning of C_{ds} (i.e., our n2.0 model), by contrast, has a dramatic impact on results, generally improving model skill.

3) Problems with the Pierson–Moskowitz spectrum in particular

If one were to take the higher wind speeds given by Moskowitz (1964) (15 m s^{-1} and higher) and simulate them using a model such as WAM or SWAN, one would find that the model is nowhere near equilibrium at the fetches and durations reported by Moskowitz (even if latent conditions in the data are accounted for). KHH tuned their model to match the Pierson–Moskowitz spectrum at a model fetch of approximately 5000 km and an infinite model duration (see KHH Fig. 8). At the fetches reported by Moskowitz, the KHH model is well below the Pierson–Moskowitz energy level (KHH's Fig. 8), and would probably be lower if their model was limited to a certain duration (e.g., 12–18 h). Thus if we assume, for the sake of argument, that there is an asymptote near which wave growth slows dramatically, and that the Pierson–Moskowitz spectrum represents this limit (an assumption of the KHH tuning process), the wave models are *still* incorrect, because they achieve this limit much too slowly. (This problem is particular to gale-force-and-higher wind speeds. At lower wind speeds given by Moskowitz, for example, approximately 10 m s^{-1}, the time and fetch required by the models to reach an asymptote is not in *clear contradiction* with the Moskowitz measurements.)

Another problem exists. These models (e.g., WAM and SWAN) scale according to U_*, while the Pierson–Moskowitz spectrum scales according to U_{10}. Thus, the models will match the Pierson–Moskowitz spectrum at one particular wind speed but will not match at any other wind speed (KHH use $U_{10} = 15$ m s^{-1} since it is near the median of the Moskowitz observations). (As an aside, we note that inspection of the Moskowitz dataset suggests that energy level is more correlated with U_{10} than with U_*. This suggests that a model with an S_{in} formulation based on U_{10} may be more accurate for modeling well-developed wave conditions at arbitrary wind speed.)

Clearly there are many unresolved questions associated with the ''limiting spectrum.'' Other researchers have conducted a detailed investigation of asymptotic limits on wave growth. See Resio et al. (1999) and Alves (2000, chapter 4) and associated references therein.

4) Alternative approach to tuning

How does one tune a stationary (i.e., duration unlimited) model to data that are generally nonstationary? We have seen that the traditional method of tuning to a theoretical asymptote in the data is fraught with problems. An alternative is to focus the tuning on shorter fetches, for example, by using the empirical relation given by Kahma and Calkoen (1992). The short-fetch data tend to be stationary, so use of a stationary model here is more easily justified.

Unfortunately, a great deal of scatter still exists between fetch-limited datasets. A model that has been tuned to match an empirical fetch-limited growth curve is only a good compromise between conflicting datasets. The scatter may be due to nonstationarity or other reasons, such as air–sea temperature differences. Perhaps the best method is to tune the models to specific, well-defined wind events, such as the Lake Michigan and Mississippi Bight cases described herein. With such simulations, temporal information is retained, and other environmental conditions such as air–sea temperature differences are well described and can be accounted for. We did not pursue tuning in our simulations, but with sufficient time and computer resources, it would be possible to do so.

f. Potential future evolution of the KHH form: A more "physics based" alternative to the $n = 2$ modification

The use of $n = 2$ in this paper is an ad hoc response to SWAN's fairly consistent underprediction of low-frequency energy and overprediction of high-frequency energy, with little or no basis in physics. Further, (as we later discovered) it is not a particularly novel approach because it was applied years ago, by Janssen et al. (1989), in the WAM model. In that earlier text, Janssen et al. point out that the basis of Hasselmann's (1974) use of $n = 1$—an assumption that the scale of the whitecap is small relative to the scale of the wave being acted on—may not be valid for the high-frequency part of the spectrum, allowing the possibility of a different dependence on wavenumber. Developing this line of thinking, we propose a more physically justified (and more novel) modification:

$$
\begin{aligned}
n &= 1, \qquad k \le k_m \\
n &= n_2, \qquad k > k_m.
\end{aligned}
\tag{18}
$$

There is no guidance for n_2. We have conducted preliminary tests with $n_2 = 2$, with promising results. Use of the "equilibrium range" dissipation term form of Phillips (1985) where $k > k_m$ is an alternative to (6). Or, in the spirit of the very tunable dissipation term of Tolman and Chalikov (1996), n_2 might be a tunable (empirical) function.

Though at first glance, it might appear that the underprediction of low-frequency energy would not be corrected by this modification (since the low-frequency n is unchanged), our preliminary tests reveal that it *is* corrected. The explanation is simple: the underprediction of low-frequency energy can be attributed to bulk parameters (e.g., mean steepness) that are influenced by the overprediction of high-frequency energy. By dissipating the high-frequency energy more, we indirectly reduce dissipation on low frequencies.

Note that though this type of modification is more physically appropriate than applying $n = 2$ everywhere in the spectrum, it unfortunately still leaves the more fundamental problem with the KHH form emphasized in this paper, namely, excessive dependence on spectrally integrated terms. Thus, this potential evolutionary development of the KHH form cannot be a final solution.

g. Other issues related to the dissipation term

Below, we discuss two issues related to the dissipation term that we do not attempt to address in this paper.

1) THE ONSET OF BREAKING

The KHH dissipation term contains a "saturation limit" on a wave spectrum insofar as the dissipation is proportional to the spectrum's integrated steepness to the fourth power. Thus, dissipation by breaking is small for immature wave conditions and rapidly becomes larger as the wave system become more developed. However, this is inconsistent with observations that in very young wave systems breaking does not occur, not even for the stochastically large waves in that system. The breaking model of Alves and Banner (2000) is an improvement in this regard, because it essentially contains a "switch" to activate breaking after a certain saturation threshold is exceeded. When this switch is turned off, dissipation is much smaller, representing weaker, nonbreaking dissipation processes (turbulence, viscosity, etc.).

2) THE EFFECT OF SWELL ON WIND SEA THROUGH THE DISSIPATION TERM

As mentioned in section 3, swell can have a nonphysical effect on the development of wind sea through the integrated steepness parameter in the KHH dissipation term (e.g., van Vledder 1999; Holthuijsen and Booij 2000). The approaches of Tolman and Chalikov (1996) and Holthuijsen and Booij (2000), which is to make the dissipation dependent only on local wind sea steepness, are advantageous in this respect.

11. Summary and conclusions

This study is motivated by a consistent underprediction of mean wave period observed in hindcasts from the third-generation wave model SWAN. This problem is caused by both underestimation of low-frequency energy and overestimation of high-frequency energy. We investigate how this bias might be corrected through modification of the model's "closure" term, steepness-limited breaking (S_{ds}). Our modification of the dissipation term consists of two parts (which we apply separately for purposes of analysis):

1) an investigation of the most suitable power (n) on the relative wavenumber term (k/k_m) in the whitecapping formulation (6) and
2) disallowing the breaking of swell, defined using a combination of wave frequency, wave age, and a local (in wavenumber space) steepness-related quantity (see section 5).

For the first item, based on considerations of the Pierson–Moskowitz spectrum, we use an alternative power of $n = 1.5$. We find that in order to achieve the asymptotic limits with the $n = 1.5$ model similar to those of the $n = 1$ model, it is necessary to increase the coefficient of proportionality [C_{ds} in (6)]. We present results from a model with $n = 1.5$ and a retuning of

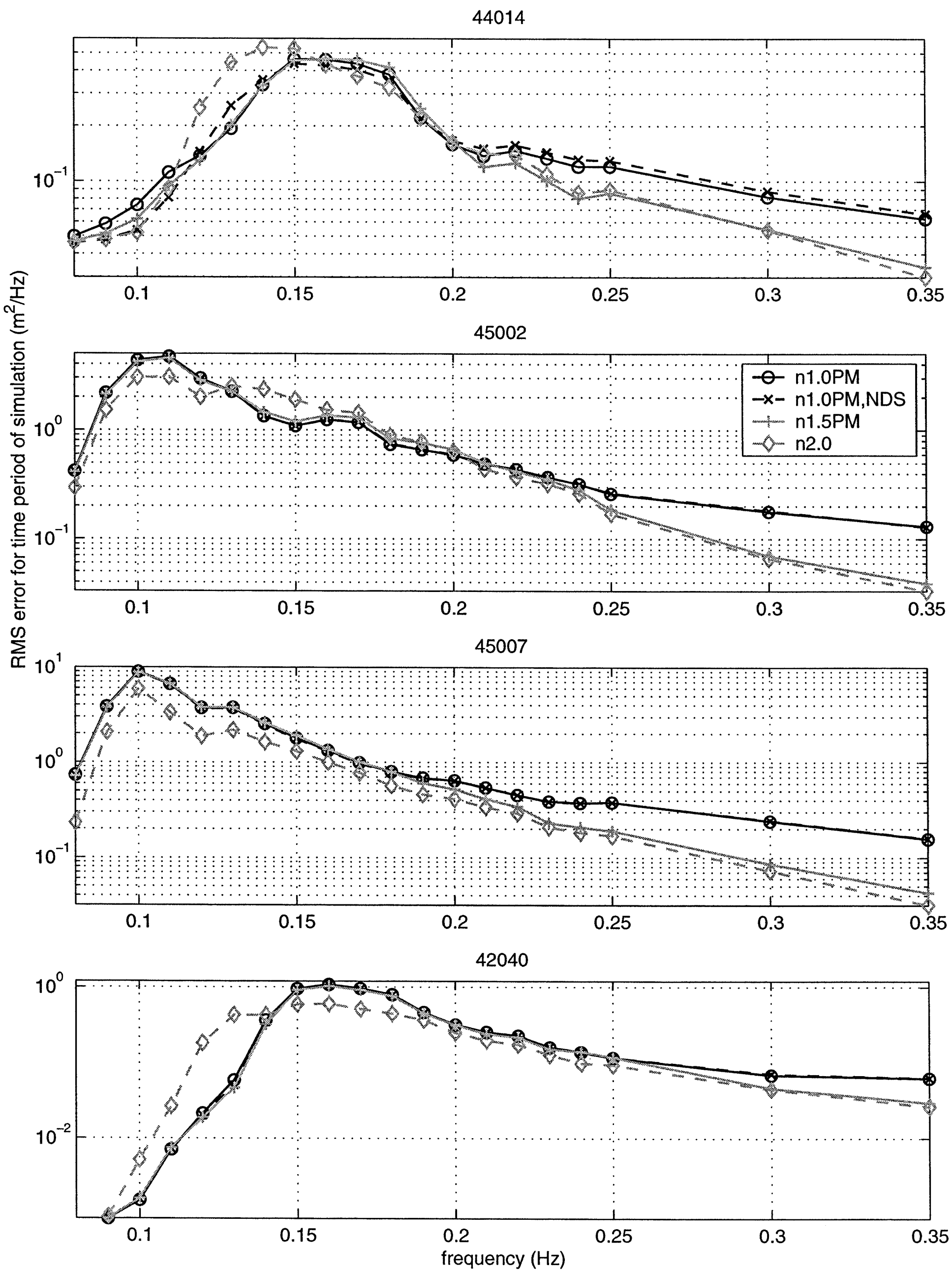

FIG. 12. Time-averaged results from the three simulations, as a function of frequency. Absolute rms errors (averaged in time over the duration of the simulations) are shown at the four NDBC buoys used in previous comparisons: SandyDuck '97 (44014), north Lake Michigan (45002), south Lake Michigan (45007), and Mississippi Bight (42040).

C_{ds}, and—to understand the implications of disregarding model behavior at extremely large fetches and durations [i.e., beyond those measured by Moskowitz (1964)]—we also present a model with $n = 2$ without any modification of C_{ds} from its original value.

Our analysis of the impact of the parameters C_{ds} and n on model skill can be considered a further development of the work by conducted by Komen et al. (1984) and Banner and Young (1994). However, our focus is on operational usage of the model and temporal effects, so our approach is different insofar as we (i) simulate specific events (which retain temporal information), as opposed to stationary canonical cases, and (ii) use a nonlinear solver that is computationally feasible for operational forecasts, as opposed to a more accurate (and more expensive) solver.

To analyze the effect of our modifications, we use three regional-scale cases [O(100 km) domain size]. For wind-forcing, we use winds measured at deep-water locations and apply these measurements to the entire domain (with interpolation as appropriate). Figure 12 summarizes the results in terms of rms error as a function of frequency (averaged in time over the duration of each simulation). Note that the SandyDuck simulation is more representative of a simulation with weaker and more complex (and therefore less certain) model forcing (wind and boundary input), while for the other two simulations, the forcing can be considered to be very accurate (and also, the wave data have much better signal-to-noise ratio than in the SandyDuck case).

The impact of modifying the parameters C_{ds} and n (modification 1 above) yields mixed results. A higher n value dramatically improves estimates of higher frequencies at all four locations. However, significant impact of a higher n is seen at the lower frequencies only if C_{ds} is not correspondingly increased (which is logical, given the form of the dissipation term). Increasing n without modifying C_{ds} dramatically improves predictions at lower frequencies for the Lake Michigan and Mississippi Bight simulations but is less accurate in the SandyDuck simulation.

Inspecting the form of the dissipation term, it is readily apparent that increasing n will directly reduce dissipation at lower frequencies. However, it is noteworthy that increasing n also *indirectly* decreases dissipation at lower frequencies by increasing the dissipation at higher frequencies, which affects integrated parameters in the dissipation function that in turn reduces overall dissipation. Investigation reveals that this indirect effect may be of equal or greater magnitude than the direct effect.

We perform a simple comparison of the four models for one canonical case to quantify differences in combined fetch- and duration-limited growth. We find that if the fetch and duration associated with the Moskowitz (1964) measurements are considered, the n2.0 model—with no tuning of C_{ds}—is (quite by accident) in better agreement with the Pierson–Moskowitz limits than is the original model.

We make no attempt to tune C_{ds} for better agreement with our three regional-scale test cases. Given that these test cases are rather limited in scope, such detailed tuning would be difficult to justify. However, we do feel that a larger set of specific, well-defined wind events, such as the Lake Michigan and Mississippi Bight cases, would provide an ideal platform for tuning of source terms (in general) preferable to the traditional method of tuning to empirical growth curves. With simulations of specific events, temporal information is retained and other environmental conditions such as air–sea temperature differences are well described and can be accounted for.

Encouragingly, significant impact of the nondissipation of swell (modification 2 above) is only seen where it should be seen, which is in the SandyDuck '97 simulation, wherein there is a low-energy swell component from the open ocean. In this case, the model's nonphysical tendency to dissipate swell during the wind event is corrected. Since the swell energy is small for the SandyDuck case, the effect of the correction on total energy level is slight. However we expect that in cases where swell energy is, by proportion, more substantial or in cases where the modeler is primarily interested in low-frequency energy, this correction can be expected to provide significantly more accurate results than the original SWAN model. This modification is also relevant to a model such as WAM, though the magnitude of the problem being corrected is smaller in the case of the WAM model.

Both of our modifications retain the basic Komen et al. (1984) form for the dissipation function. With the modifications, a model using this dissipation form becomes more suitable for general application and likely more skillful in a large majority of cases, given accurate forcing. However, our decision to retain this dissipation form should not be interpreted as an endorsement. In fact, based on our experiences in this study, we do not feel that this form should be a permanent fixture of WAM-type models. It contains too many apparent deficiencies [for example, various problems resulting from excessive reliance on spectrum-integrated parameters and also problems mentioned in Banner and Young (1994)]. If model skill is the objective, then a very tunable, empirical approach such as Tolman and Chalikov (1996) is probably the best near-term solution. However, a greater devotion to the physics of gravity waves may be the best approach for the long term. For this to bear fruit however, we feel that there needs to be revolutionary (not evolutionary) development of the manner in which we represent phase-associated processes (such as whitecapping) in stochastic models.

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APPENDIX

Model Options and Controls Used in the Hindcasts Presented

Case	Lake Michigan	Mississippi Bight	SandyDuck
Nonstationary?	Yes	Yes	Yes
Δt	10 min	10 min	10 min
nx	127	81	151 (outer nest)
Δx	2000 m	3630 m	2000 m (outer nest)
ny	249	41	261 (outer nest)
Δy	2000 m	4720 m	2000 m (outer nest)
$n\theta$	36	36	36
$\Delta\theta$	10°	10°	10°
$n\sigma$	34	36	34
σ_l (lowest modeled frequency)	0.07 Hz	0.08 Hz	0.05 Hz
σ_h (highest modeled frequency)	1.00 Hz	1.00 Hz	1.00 Hz
Wind forcing?	Yes (from two buoys)	Yes (from one buoy)	Yes (from three buoys)
Boundary forcing?	No (entire lake included)	No (insignificant amount of swell during this period)	Yes (swell only; stationary)
Initial condition	Rest	Rest	Produced using stationary computation with no winds (boundary forcing only) to fill domain with ''background swell''
Bottom friction	SWAN default (JONSWAP)		
Triads?	Yes (default settings for SWAN triads formulation), but did not affect results presented		
Numerics	Default for v40.01		
S_{in} term	SWAN default (WAM3)		
No. of iterations per time step	SWAN default (1, i.e. no iterations)		

Other options/features:

1) for the Lake Michigan case, higher-order numerics (no effect)
2) for the Lake Michigan case, smaller time step (no effect)
3) for the SandyDuck case, a nested (more high resolution) case was run, but this increased resolution did not affect results appreciably

Variations of whitecapping formulation:

1) SWAN default ($n = 1.0$, $C_{ds} = 2.36 \times 10^{-5}$) ($C_{ds}$ tuned to Pierson–Moskowitz by KHH) (denoted n1.0PM in the text)
2) Increased n, C_{ds} tuned to Pierson–Moskowitz in this study ($n = 1.5$, $C_{ds} = 4.5 \times 10^{-5}$) (denoted n1.5PM in the text)
3) Increased n, C_{ds} not altered ($n = 2.0$, $C_{ds} = 2.36 \times 10^{-5}$) (denoted n2.0 in the text)
4) SWAN default ($n = 1.0$, $C_{ds} = 2.36 \times 10^{-5}$), with no breaking of swell, as defined by criterion (11) (denoted n1.0PM,NDS in the text)

REFERENCES

Alves, J. H. G. M., 2000: A saturation-dependent dissipation source function for wind-wave modeling applications. Ph.D. thesis, The University of New South Wales, Sydney, Australia, 241 pp.

——, and M. L. Banner, 2000: A saturation-dependent dissipation source term for wind-wave modeling applications: Fetch-limited case. *Proc. 27th Int. Conf. on Coastal Engineering*, Sydney, Australia, ASCE, 269–282.

Banner, M. L., and I. R. Young, 1994: Modeling spectral dissipation in the evolution of wind waves. Part I: Assessment of existing model performance. *J. Phys. Oceanogr.,* **24,** 1550–1571.

Bender, L. C., 1996: Modification of the physics and numerics in a third-generation ocean wave model. *J. Atmos. Oceanic Technol.,* **13,** 726–750.

Booij, N., R. C. Ris, and L. H. Holthuijsen, 1999: A third-generation wave model for coastal regions. 1. Model description and validation. *J. Geophys. Res.,* **104,** 7649–7666.

Donelan, M. A., and W. J. Pierson, 1987: Radar scattering and equilibrium ranges in wind-generated waves with application to scatterometery. *J. Geophys. Res.,* **92,** 4971–5029.

——, and Y. Yuan, 1994: Wave dissipation by surface processes. *Dynamics and Modelling of Ocean Waves*, G. J. Komen et al., Eds., Cambridge University Press, 143–155.

Hasselmann, K., 1974: On the spectral dissipation of ocean waves due to whitecapping. *Bound.-Layer Meteor.,* **6,** 107–127.

——, and Coauthors, 1973: Measurements of wind wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). Herausgegeben vom Deutsch. Hydrograph. Institut., Reihe A, No. 12, 95 pp.

Hasselmann, S., and K. Hasselmann, 1985: Computations and parameterizations of the nonlinear energy transfer in a gravity-wave spectrum. Part I: A new method for efficient computations of the exact nonlinear transfer integral. *J. Phys. Oceanogr.,* **15,** 1369–1377.

——, ——, J. H. Allender, and T. P. Barnett, 1985: Computations and parameterizations of the linear energy transfer in a gravity wave spectrum. Part II: Parameterizations of the nonlinear transfer for application in wave models. *J. Phys. Oceanogr.,* **15,** 1378–1391.

Holthuijsen, L. H., and N. Booij, 2000: Oceanic and near-shore whitecapping effects in SWAN. *Proc. Sixth Int. Workshop on Wave Hindcasting and Forecasting,* Monterey, CA, Meteorological Service of Canada, 362–368.

Hsu, Y. L., W. E. Rogers, J. M. Kaihatu, and R. A. Allard, 2000: Application of SWAN in the Mississippi Sound. *Proc. Sixth Int. Workshop on Wave Hindcasting and Forecasting,* Monterey, CA, Meteorological Service of Canada, 398–403.

Hwang, P. A., D. W. Wang, E. J. Walsh, W. B. Krabill, and R. N. Swift, 2000: Airborne measurements of the wavenumber spectra of ocean surface waves. Part I: Spectral slope and dimensionless spectral coefficient. *J. Phys. Oceanogr.,* **30,** 2753–2767.

Janssen, P. A. E. M., 1989: Wave induced stress and the drag of air flow over sea waves. *J. Phys. Oceanogr.,* **19,** 745–754.

——, 1991: Quasi-linear theory of wind-wave generation applied to wave forecasting. *J. Phys. Oceanogr.,* **21,** 1631–1642.

——, 1992: Consequences of the effect of surface gravity wave on the mean air flow. *Breaking Waves: IVTAM Symposium, Sydney, Australia, 1991,* M. L. Banner and R. H. J. Grimshaw, Eds., Springer-Verlag, 193–198.

——, P. Lionello, and L. Zambresky, 1989: On the interaction of wind and waves. *Philos. Trans. Roy. Soc. London,* **A329,** 289–301.

——, H. Günther, S. Hasselmann, K. Hasselmann, G. J. Komen, and L. Zambresky, 1994: Simple tests. *Dynamics and Modelling of Ocean Waves,* G. J. Komen et al., Eds., Cambridge University Press, 244–257.

Kahma, K. K., and C. J. Calkoen, 1992: Reconciling discrepancies in the observed growth of wind-generated waves. *J. Phys. Oceanogr.,* **22,** 1389–1405.

Kitaigorodskii, S. A., and Y. Z. Miropolskii, 1968: Dissipation of turbulent energy in the surface layer of the ocean. *Izv. Acad. Sci. USSR, Atmos. Oceanic Phys.,* **4,** 647–659.

Komen, G. J., S. Hasselmann, and K. Hasselmann, 1984: On the existence of a fully developed wind-sea spectrum. *J. Phys. Oceanogr.,* **14,** 1271–1285.

——, L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann, and P. A. E. M. Janssen, 1994: *Dynamics and Modelling of Ocean Waves.* Cambridge University Press, 532 pp.

Monbaliu, J., R. Padilla-Hernández, J. C. Hargreaves, J. C. C. Albiach, W. Luo, M. Sclavo, and H. Günther, 2000: The spectral wave model, WAM, adapted for applications with high resolution. *Coastal Eng.,* **41,** 41–62.

Moskowitz, L., 1964: Estimates of the power spectrums for fully developed seas for wind speeds of 20 to 40 knots. *J. Geophys. Res.,* **69,** 5161–5179.

Phillips, O. M., 1985: Spectral and statistical properties of the equilibrium range in wind-generated gravity waves. *J. Fluid Mech.,* **156,** 505–531.

Pierson, W. J., and L. Moskowitz, 1964: A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii. *J. Geophys. Res.,* **69,** 5181–5190.

Resio, D. T., and W. Perrie, 1991: A numerical study of nonlinear energy fluxes due to wave–wave interactions. I. Methodology and basic results. *J. Fluid Mech.,* **223,** 609–629.

——, V. R. Swail, R. E. Jensen, and V. J. Cardone, 1999: Wind speed scaling in fully developed seas. *J. Phys. Oceanogr.,* **29,** 1801–1811.

Ris, R. C., 1997: Spectral modeling of wind waves in coastal areas. Ph.D. thesis, Delft University of Technology, 160 pp.

Rogers, W. E., P. A. Hwang, D. W. Wang, and J. M. Kaihatu, 2000: Analysis of SWAN model with in situ and remotely sensed data from SandyDuck '97. *Proc. 27th Int. Conf. on Coastal Engineering,* Sydney, Australia, ASCE, 812–825.

Snyder, R. L., F. W. Dobson, J. A. Elliott, and R. B. Long, 1981: Array measurement of atmospheric pressure fluctuations above surface gravity waves. *J. Fluid Mech.,* **102,** 1–59.

Stelling, G. S., and J. J. Leendertse, 1992: Approximation of convective processes by cyclic AOI methods. *Proc. Second Int. Conf. on Estuarine and Coastal Modeling,* Tampa, FL, ASCE, 771–782.

Tolman, H. L., 1991: A third generation model for wind waves on slowly varying, unsteady and inhomogeneous depths and currents. *J. Phys. Oceanogr.,* **21,** 782–797.

——, and D. Chalikov, 1996: Source terms in a third-generation wind wave model. *J. Phys. Oceanogr.,* **26,** 2497–2518.

van Vledder, G. Ph., 1999: Source term investigation: SWAN. Rev. 2, Rep. A162R1r2, Alkyon, 83 pp. plus figures.

——, T. H. C. Herbers, R. E. Jensen, D. T. Resio, and B. Tracy, 2000: Modeling of nonlinear quadruplet wave–wave interactions in operational models. *Proc. 27th Int. Conf. on Coastal Engineering,* Sydney, Australia, ASCE, 797–811.

WAMDI Group, 1988: The WAM model—A third generation ocean wave prediction model. *J. Phys. Oceanogr.,* **18,** 1775–1810.

Yuan, Y., C. C. Tung, and N. E. Huang, 1986: Statistical characteristics of breaking waves. *Wave Dynamics and Radio Probing of the Ocean Surface,* O. M. Phillips and K. Hasselmann, Eds., Plenum, 265–272.