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Refinements to an Optimized Model-Driven Bathymetry Deduction Algorithm

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Abstract: In this study we describe a numerical algorithm which deduces characteristics of the bottom bathymetry given free surface elevation records dense in space but sparse in time. The method makes use of the Levenberg-Marquardt numerical optimization scheme in conjunction with a time-domain nonlinear model. Iteration occurs until the mismatch between the free surfaces of the data and model are minimized; the bathymetry is adjusted in order to achieve this minimum. The sensitivity measure is a by-product of the calculation, and determines the invertibility of the system. Due to robustness concerns, we limit ourselves to deduction of bathymetric profile parameters. Tests of the system using monochromatic, irregular and groupy waves show favorable results; the latter is particularly notable give the difficulty standard inversion methods have had with groupy waves. A two-stage system is also outlined, in which a simple parameterization for a nearshore bar is developed and utilized. The first stage determines the mean profile, while the second stage determines the bar characteristics using the first stage results as the initial iterate. To extend the method's capabilities further, the use of phase speed records is discussed.

Introduction

Nearshore wave and circulation models have become much more sophisticated in both structure and physics. However, the accuracy of these models is highly dependent on the quality and resolution of the nearshore bathymetry. Direct measurement of the bathymetry is difficult under the most benign conditions. Additionally, the nearshore waves and currents obviously react to the changes in the bathymetry in deterministic ways. The physical relationships between free surface evolution and bathymetric variations (represented in numerical models of wave evolution) can be exploited to deduce the bathymetry given realizations of the free surface or other properties (e.g., phase speeds, etc.).

Concurrently, remote sensing capabilities have also undergone significant development. Synthetic aperture radar (SAR), video, electro-optical sensors. and X-band (marine) radar are capable of high resolution measurements of light intensity from

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the free surface. Wave kinematics can thus be calculated by linking variations in intensity to phase lines of the waves. Direct sensing of the free surface or surface currents can be accomplished using airborne lidar (for a single snapshot of the free surface) or interferometric synthetic aperture radar (INSAR). These latter systems will give either a direct free surface measurement (lidar) or an inferred measurement from the currents (INSAR). The challenge, therefore, lies in incorporating these data into bathymetry deduction algorithms.

The problem of bathymetry deduction from remotely-sensed wave characteristics is one that has been a military focus since World War II, but has received much attention recently. These recent developments can be roughly classified into two areas. The first approach focuses on gleaning wave kinematics and dynamics from the free surface imagery, then using the linear dispersion relation:

$$\omega^2 = gk \tanh kh \qquad (1)$$

in which ω is the radian wave frequency, g is gravitational acceleration, and k is the wavenumber ($=2\pi/L$, where L is the wavelength). Bell (1999) used phase speeds calculated from X-band radar imagery and Equation (1) to infer the bathymetry, with favorable results. Holland (2001) performed a similar analysis using phase speeds from video data, while Dugan et al (2001) utilized kinematics from airborne electro-optical systems.

The second approach combines data and numerical models into a numerical optimization problem, in which the total mismatch between the data and the model results is iteratively minimized, with the bathymetry correspondingly adjusted. Kennedy et al. (2000) and Narayanan and Kaihatu (2000) are examples of this approach. We also note that Grilli (1997) created nomographs for direct calculation of the bathymetry from phase speeds based on results from his fully nonlinear boundary element model.

Our development uses the second approach, and is a refinement of that detailed in Narayanan and Kaihatu (2000) in that we are now able to "zoom in" on nearshore features such as sandbars. Additionally, we explicitly investigate the improvement of this model over direct inversion of Equation (1) with regards to wave groups. We conclude with considerations of future directions for this work.

Numerical Models and Data

In the interest of computational efficiency during the initial development of the bathymetric deduction algorithm, we used the Korteweg-deVries (KdV) equation (Korteweg and deVries 1895) as the wave model. Throughout this study, we will be using output from the KdV model as synthetic data for system testing; any physical shortcomings contained in the wave model is therefore reflected in the data as well, and so thus is consistent. The modular nature of the completed algorithm is such that any wave model can be used instead of the KdV model.

We assume that the free surface imagery is in the form of "snapshots" of the free surface, dense in space but well-separated in time. This sort of free surface data stream would be akin to having a lidar-equipped aircraft make several sweeps over the same area. This is in fact a severe test of the algorithm, since the time span between snapshots is greater than a dominant wave period. The likelihood is great that a more generous data set will yield better results for the bathymetry.

Bathymetric Parameterizations and Optimization Techniques

The numerical optimization technique used herein is the Levenberg-Marquardt method, which is an overrelaxed nonlinear least squares technique (see Press et al. 1986 for sample algorithms). This method has been used in many applications, and excels in the case of highly nonlinear optimization with multiple local minima and elongated, shallow error surfaces.

At the time of writing, the authors are preparing a journal manuscript on the topic at hand; the full technique is explained in an appendix of the manuscript. In the interest of brevity, we simply write the matrix equation to be solved:

$$\Delta h = (\mathbf{A}^T \mathbf{A} + \mu^2 \mathbf{I}) \mathbf{A}^T \Delta \eta \qquad (2)$$

where Δh is a vector containing the difference between the prior and new iterates of water depth; $\mathbf{I}$ is an identity matrix, $\Delta \eta$ is a matrix of the data-model mismatches, μ is an over-relaxation parameter, and the matrix $\mathbf{A}$ is known as the "sensitivity" matrix:

$$A = \begin{pmatrix} \frac{\partial \eta_1}{\partial h_1} & \frac{\partial \eta_1}{\partial h_2} & \cdots & \frac{\partial \eta_1}{\partial h_m} \\ \frac{\partial \eta_2}{\partial h_1} & \frac{\partial \eta_2}{\partial h_2} & \cdots & \frac{\partial \eta_2}{\partial h_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \eta_n}{\partial h_1} & \frac{\partial \eta_n}{\partial h_2} & \cdots & \frac{\partial \eta_n}{\partial h_m} \end{pmatrix} \qquad (3)$$

given m total parameters (water depths to be determined, in this case) and n total data points. These data could be distributed in space (as is the case for wave staffs measuring time series at discrete locations) or time (remotely sensed snapshots of the free surface taken at discrete times). Multiplying by the transpose creates an mxm square matrix. The diagonal elements of this square matrix comprise an $mx1$ vector:

$$diag(A^T A) = \left[\sum_{i=1}^{n} \left(\frac{\partial \eta_i}{\partial h_1} \right)^2 \quad \sum_{i=1}^{n} \left(\frac{\partial \eta_i}{\partial h_2} \right)^2 \quad \cdots \quad \sum_{i=1}^{n} \left(\frac{\partial \eta_i}{\partial h_m} \right)^2 \right] \qquad (4)$$

Each element of this vector is known as a sensitivity measure, and is a key to the solvability of the system. In general, the relative amplitudes of all the m sensitivity measures must be within three orders of magnitude in order for the system to be invertible and the parameters identifiable. This insures that the search for any one parameter does not dominate the iteration process. The sensitivity measures are not the only considerations present in the inversion process. However, the sensitivity measures are also useful in other aspects of the inversion problem, such as array design and the formulation of data sampling strategies. We will discuss this aspect further in a later section.

During the initial development of this system, we endeavored to be able to find an arbitrary number of water depths in our domain, at least as many as we had grid points in the wave model. However, difficulties with nonconvergence caused us to consider using various parameterized forms of the bathymetric cross-shore profile, with the various free parameters within the assumed forms (rather than the depths themselves) becoming the targets of the search. We considered several parameterized forms, including Dean's equilibrium beach profile (Dean 1977), exponential forms (Bodge 1992), and polynomial forms. We determined that both Dean's equilibrium beach profile and exponential forms worked best with the inversion scheme detailed here. In a later section we describe how the ostensibly-limiting form for bathymetry can be used as an initial guess for more detailed inversions in the nearshore.

Monochromatic and Random Waves

Our tests of the system with both monochromatic and irregular waves have been detailed in Narayanan and Kaihatu (2000). We used Dean's equilibrium beach profile form $h(x)=Ax^m$, and use the method described in the previous section to determine the free parameters A and m. A set of free surface snapshots would be generated by propagating the waves over bathymetry for which the parameters are known, then sampling the modeled free surface elevations over the entire domain at three discrete points in time (t=16, 20 and 24 seconds). The domain was 35 meters long, and the wave had a peak period of 5 seconds. The bathymetric parameters were then varied and the iteration procedure begun; each step of the iteration yielded trial values for the bathymetric parameters A and m. Convergence of the system to the actual parameter values was highly dependent on initial guess. Plots of the results for the random wave case can be seen in Narayanan and Kaihatu (2000). An empirically-derived map of convergence as a function of initial position in parameter space is shown in Figure 1. The plot reveals that initial guesses which tend to overestimate the offshore depth (high values of A) tend to lead to convergence more than those parameter values which underestimate the offshore depth.

Wave Groups

In our investigations into the applicability of direct inversion methods utilizing the linear dispersion relation (Equation 1), it was found that the case of wave groups required special treatment. Both monochromatic and random wave trains have free surface undulations which are more or less strongly coupled to variations in the bathymetry. Strong wave groups (those comprised of a small number of frequency components) have free surface characteristics which may be more due to the interference patterns inherent in the group rather than bathymetric variations. This feature tends to cause underestimation of wavenumbers and results in overprediction of the water depth.

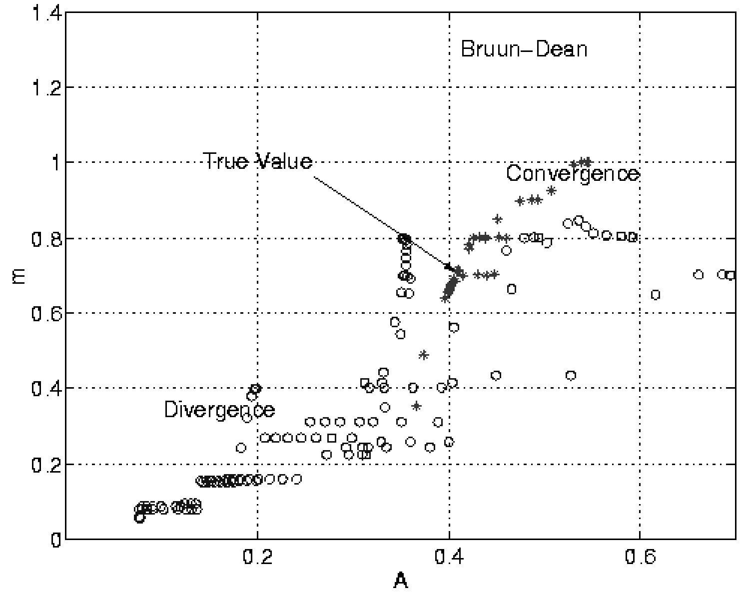

Figure 1. Empirical convergence map, Dean's equilibrium beach profile under random waves. Initial guesses of *A* and *m* corresponding to open circles will diverge, while those corresponding to asterisks will converge.

Consideration of additional data in time could be expected to improve results. This is true of random waves, since the peak spectral values gleaned from the data and used in the linear dispersion relation for inversion would be more statistically representative of the actual wave field. However, this does not apply to strong wave groups. Figure 2 shows the results of a numerical experiment detailed in Kaihatu et al. (1999), in which a wave model was used to propagate random and groupy waves over a bathymetric cross-shore transect measured at the US Army Field Research Facility at Duck, NC. Over 2000 temporal realizations of the free surface (amounting to around 200 seconds of time) were generated. A wavelet transform was used to glean wavenumbers for each point along the cross-shore transect for each temporal realization, then averaged across all realizations. This gave a statistically significant peak wavenumber for each point along the cross-shore transect. This, along with the peak frequency from the incident spectrum, was used with the linear dispersion relation to solve for the water depth. Figure 2a shows the case for random waves; despite the crudity of the inversion method, the depths match quite well. Figure 2b shows the result of the same analysis using wave groups. Despite the significant temporal averaging, the strong group signature is still apparent in the result.

The inversion procedure described earlier was then applied to the case of wave groups. Results are shown in Figure 3. It is apparent that the procedure works quite well. Figure 3a shows the initial and true profiles under wave groups, while Figure 3b shows the empirical convergence map. There appears to be a cluster of convergence near the actual value of *A* and *m* within which initial guesses for *A* and *m* will lead to convergence. The success of the procedure for wave groups indicates that the benefit of this method is the lack of wave kinematics or other assumptive properties that require extraction from imagery.

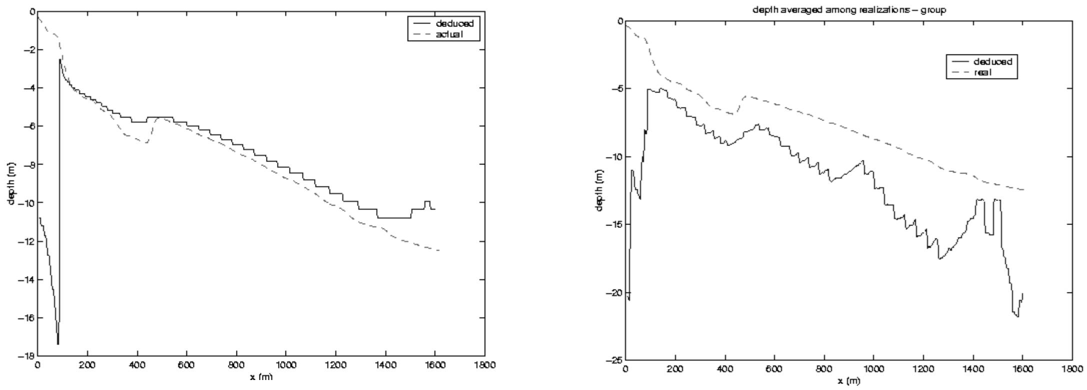

Figure 2. Results of direct depth inversion using linear dispersion relations and wave kinematics gleaned from numerical results by wavelet transforms. Solid line: estimated bathymetry from analysis. Dashed line: actual bathymetry from Duck, NC. Left (a): Random waves. Right (b): Wave groups.

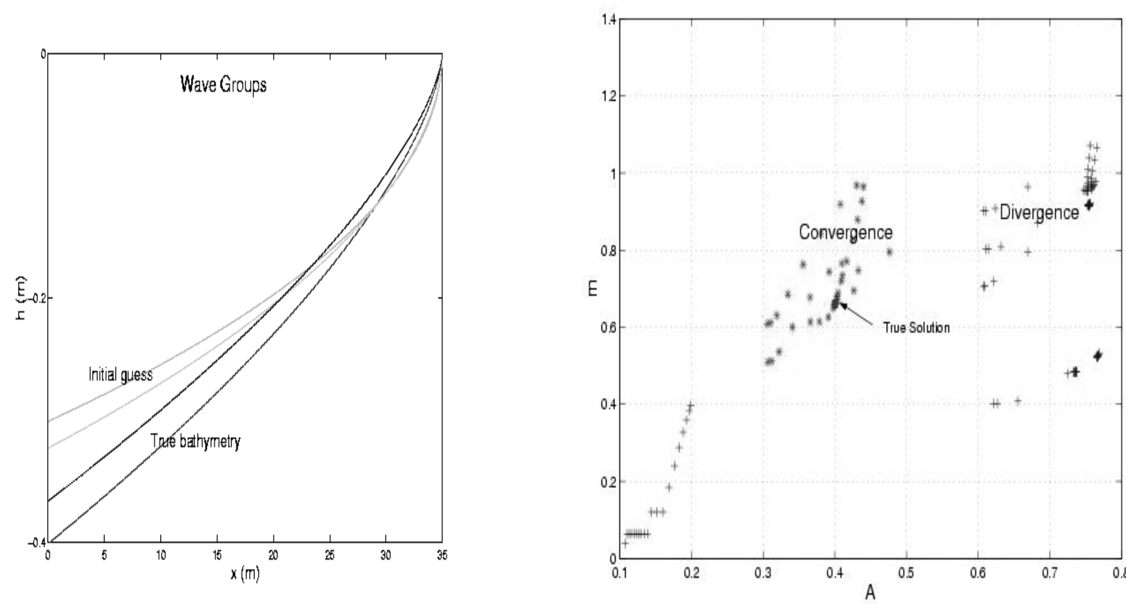

Figure 3. Results of bathymetric inversion using wave groups. Left (a): Initial and final bathymetric profiles. Right (b): Empirical convergence map

Two-Stage Inversion

We further refined the bathymetric inversion scheme by adding a second stage to the iteration routine. This second stage uses the estimation of the mean profile as the first guess for a second round of iterations.

As with the single stage inversions described earlier, we use a parameterized bathymetry for the second stage (as with the previously-described inversion, we encountered convergence problems when we attempted to use every water depth in our numerical grid as a parameter for which to search). Our parameterization of this second stage bathymetry includes a simplistic formulation to account for the bar:

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$$h_2(x) = \alpha \sin \beta(x - x_0) \quad (5)$$

where the subscript “2” denotes the results of the second stage searchm α and β are shape factors for the bar, and x_0 is the location of the offshore edge of the bar. In theory one could search for all five parameters in one stage; however, a sensitivity analysis had determined that the search for the bar parameters must be performed over a much smaller section of the domain than that of the mean profile parameter search. Thus, the search takes place in two sets of iterations. The first set of iterations yields the mean profile parameters *A* and *m*, and the second search seeks the other three parameters.

We tested the two-stage inversion system with bathymetry at Duck, NC. This particular set of bathymetry was collected during the October 1990 DELILAH experiment, during which a bar was almost always present. A TMA-type wave spectrum was run over this bathymetry and the free surface information along a cross-shore transect of the domain stored at t=16, 20 and 24 seconds. This data then seeded the inversion. Figure 4 shows the result of the two-stage inversion. The first stage parameters do well in replicating the mean profile at Duck for much of the domain, and even capture the mean of the bar fairly well. The second stage results (shown in the extreme nearshore area of the domain) captures some of the reverse curvature of the bar. As mentioned before, the sensitivity analysis for this scenario indicated that the offshore extent of the second stage iterations needed to be very close to shore if the inversion was to be sufficiently robust. This makes intuitive sense, since one would not expect information well offshore of the bar to be essential to the determination of bar parameters; however, it is reassuring that a quantitative sensitivity analysis confirms a qualitative observation.

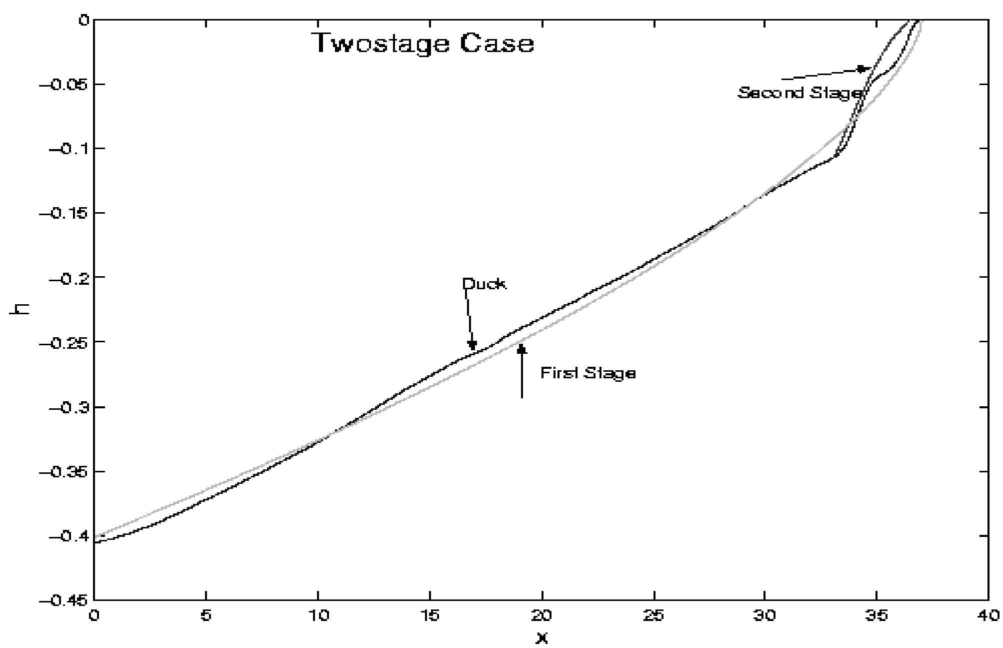

Figure 4. Results of two-stage inversion process: comparison to Duck bathymetry.

Phase speed

Though the overall approach described herein is powerful and has wide potential applications, there are several obvious disadvantages with our specific application. The reliance on bathymetric parameterization is a severe limitation, and would hamper an extension to longshore variations of bathymetry. Additionally, the use of temporally sparse, spatially dense free surface elevations as data limits potential applicability of the system to a small range of remote sensing instrumentation (interefereometric SAR, lidar). Adaptation of the inversion algorithm to phase speed measurements would greatly expand the possible remote sensing platforms which can be used in tandem, as mentioned earlier.

It can be somewhat cumbersome to calculate phase speeds from a free surface time domain wave model, though correlation algorithms such as those used by Bell (1999) and Holland et al. (2001) can be used. However, it is more expedient to use a frequency-domain model for this application, since it is simpler to calculate phase speed, even with nonlinear effects.

To investigate the efficacy of calculating phase speed from a frequency domain model, we use the model of Kaihatu and Kirby (1995), with extensions by Kaihatu (2001), to evolve a wave spectrum over a planar bathymetry. This model is developed in terms of the complex amplitude of the free surface, with resonant triad interactions representing the shallow water wave nonlinearity. The particular advantages of this model are addressed in Kaihatu and Kirby (1995). Substituting a real-valued amplitude and phase in place of the complex amplitude, then retaining the imaginary part of the resulting equation, yields a direct expression for the phase speed. We initialize the model with a TMA-type spectrum (peak period of X seconds and a gamma of Y), and run the spectrum over a sloping bathymetry with an offshore depth of 8 meters and a bottom slope of 0.0005. Figure 5 shows the nonlinear phase speed calculation from the model (using the peak frequency), with a corresponding calculation for linear phase speed. The nonlinear phase speed is greater than that from linear theory for the offshore part of the domain. As breaking begins to dominate, the waveheight at the peak frequency becomes smaller and nonlinear effects are reduced. If video data is utilized, the temporal statistics (due to the long measurement time) can approach those of the model, thus allowing a one-to-one correspondence (in theory) between average characteristics from the model and those from the data. Thus phase speed estimates would be representative of the same amount of time, lending confidence to our use of phase speed for depth inversion.

Summary

We have outlined a method for determining water depth characteristics from remotely sensed data. In contrast to earlier data-intensive methods, we use numerical optimization and a numerical wave model to obtain water depth estimates. The numerical technique used (Levenberg-Marquardt optimization) is well-suited to the potential nonuniqueness of the inversion process.

Difficulties with convergence compelled us to utilize bottom profile parameterization as a means for obtaining bathymetric characteristics. We obtained excellent results using free surface elevations gleaned from the wave model as “data,” and using the numerical algorithms to iterate on the bathymetric parameters. This was true for monochromatic, random, and groupy waves. The latter case is notable in that straightforward depth inversion using the linear dispersion relation (Equation 1) proved to be problematic since the free surface characteristics are more a function of the interference patterns of the waves in the group than the bathymetry variations.

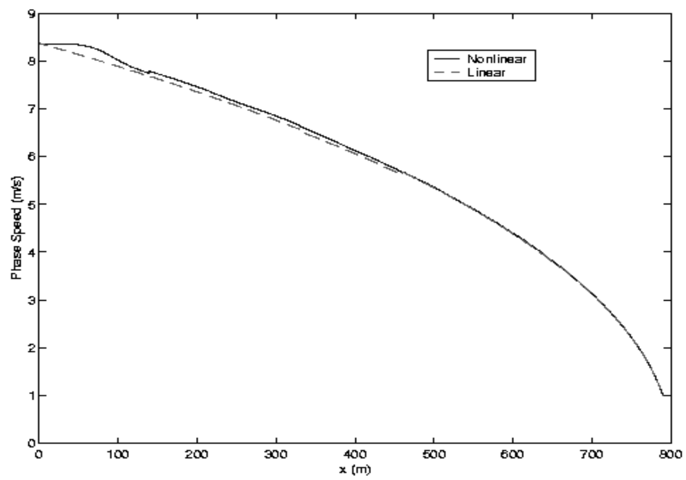

Figure 5. Phase speed estimates from the nonlinear frequency domain model of Kaihatu and Kirby (1995). Phase speed calculated from the peak frequency of a wave spectrum shoaled over a sloping bottom. Solid line: linear phase speed. Dashed line: nonlinear phase speed.

We then also investigated the parameterization of a nearshore bar in order to extend the original algorithm by implementing a two-stage process. The mean profile parameters would be obtained from the first stage, and then used as an initial iterate for the second stage, which would yield the bar parameters. We tested this two-stage system using a bathymetric transect from Duck, NC. Both the mean profile and the nearshore bar parameters matched the actual profile fairly closely.

In the algorithm testing we assumed that our data consisted of spatially dense snapshots of the free surface elevations at discrete points in time. This focus limits the applicability of the system to a small set of remote sensing instrumentation. In order to overcome this limitation, we intend to focus on the use of phase speeds from both the model and data to perform the inversion. Phase speed estimates are obtainable from many more remote sensing platforms than free surface elevations, and also exhibit less variability than free surface elevations; thus we can expect a more flexible and robust algorithm. This will be the focus of the next phase of this research.

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