Average Seasonal Circulation in the Northern Gulf of Mexico

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ABSTRACT:

Velocity measurements from moorings, shipboard acoustic Doppler current profilers (ADCPs), and satellite tracked drifters are used to estimate the average seasonal circulation along the continental shelf in the northern Gulf of Mexico. In addition to velocity measurements, sea surface height (SSH) information from the TOPEX/Poseidon satellite is also included in the seasonal flow field estimations. The northern Gulf of Mexico is split up into two regions, the Northeast Gulf of Mexico (NEGOM) and the Louisiana-Texas Shelf (LATEX). The circulation within these two regions are estimated individually. A weighted least squares technique is used to combine these data sets with a system of dynamics, continuity and momentum. Momentum is governed by the linear, time-independent, shallow water equations and is forced by wind stress (provided by the Navy Global Ocean and Atmospheric Prediction System). In addition to continuity and momentum, boundary and smoothing constraints are included in the dynamics to restrict the flow that is perpendicular to the coast and to remove features that are not resolved by the discretized grid or measurements. The solution weights are based on the expected errors to the dynamical equations and measurements. A conjugate gradient inverse method determines the flow field that best satisfies the weighted data and dynamics for each season.

DYNAMICS:

Continuity: $\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$ Boundary: $Un_x+Vn_y=0$

Momentum:

$$fV-gH\frac{\partial\eta}{\partial x}+K_H\left(2\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 V}{\partial x\partial y}\right)-C_D|\vec{u}|\frac{U}{H}=-\frac{\tau_x}{\rho_0}$$

$$-fU-gH\frac{\partial\eta}{\partial y}+K_H\left(\frac{\partial^2 V}{\partial x^2}+2\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 U}{\partial x\partial y}\right)-C_D|\vec{u}|\frac{V}{H}=-\frac{\tau_y}{\rho_0}$$

Smoothing: $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial x\partial y}=0$ $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial x\partial y}=0$ $\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}+\frac{\partial^2\eta}{\partial x\partial y}=0$

U, V, and η are the seasonally averaged vertically integrated velocity components and SSH respectively that are being solved for at each grid point.

$K_H=1000\,\mathrm{m^2/sec}$, $C_D=0.002$, H, and $|\vec{u}|$ are the horizontal eddy viscosity coefficient, bottom drag coefficient, bathymetry, and average magnitude of bottom velocity.

τ_x, τ_y, n_x, and n_y are the components of wind stress and the vector perpendicular to the coastline respectively.

SOLUTION METHOD:

Construction of Weighted Least Squares:

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\vec{x}=\mathbf{A}^T\mathbf{W}\vec{b}$$

$$\underbrace{\begin{bmatrix}\mathbf{A}_{Cont}\\ \mathbf{A}_{Mom}\\ \mathbf{A}_{Bnd}\\ \mathbf{A}_{Smth}\\ \mathbf{A}_{Mem}\end{bmatrix}}_{\text{coefficient matrix}}\underbrace{\begin{bmatrix}U\\ V\\ \eta\end{bmatrix}}_{\text{state vector}}\underbrace{\begin{bmatrix}\mathbf{W}_{Cont} & 0 & 0 & 0 & 0\\ 0 & \mathbf{W}_{Mom} & 0 & 0 & 0\\ 0 & 0 & \mathbf{W}_{Bnd} & 0 & 0\\ 0 & 0 & 0 & \mathbf{W}_{Smth} & 0\\ 0 & 0 & 0 & 0 & \mathbf{W}_{Mem}\end{bmatrix}}_{\text{weighing matrix}}\underbrace{\begin{bmatrix}\vec{b}_{Cont}\\ \vec{b}_{Mom}\\ \vec{b}_{Bnd}\\ \vec{b}_{Smth}\\ \vec{b}_{Mem}\end{bmatrix}}_{\text{forcing vector}}$$

$$\mathbf{W}_{Mem}=\frac{1}{n\langle er\rangle^2}\mathbf{I}_{(n\times n)}$$

Estimated expected errors $\langle er\rangle$ of dynamics:

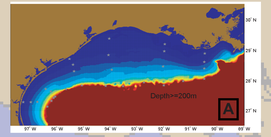
continuity	momentum	smoothing of U and V	smoothing of η	boundary
$6.1\times10^{-9}\,\mathrm{m/sec}$	$2.5\times10^{-5}\,\mathrm{m^2/sec^2}$	$7.1\times10^{-8}\,\mathrm{1/sec}$	$3.1\times10^{-10}\,\mathrm{1/m}$	$7.1\times10^{-2}\,\mathrm{m^2/sec}$

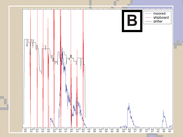
Estimated expected errors $\langle er\rangle$ of measurements:

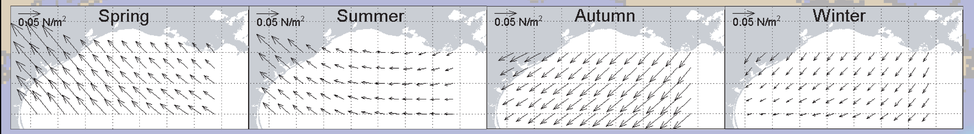
	mooring data	shipboard ADCP data	SCULP drifter data	TOPEX data
LATEX:	$2.2\,\mathrm{m^2/sec}$	$9.3\,\mathrm{m^2/sec}$	$13.7\,\mathrm{m^2/sec}$	1.3cm
NEGOM:	$3.2\,\mathrm{m^2/sec}$	$8.3\,\mathrm{m^2/sec}$	$9.1\,\mathrm{m^2/sec}$	2.0cm

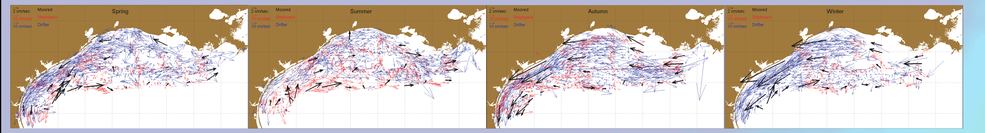
A conjugate gradient elliptic solver with a red-black SSOR preconditioner is used to minimize the cost function $(\mathbf{A}\vec{x}-\vec{b})^T\mathbf{W}(\mathbf{A}\vec{x}-\vec{b})$ iteratively, and determine the solution that best satisfies the weighted least squares problem.

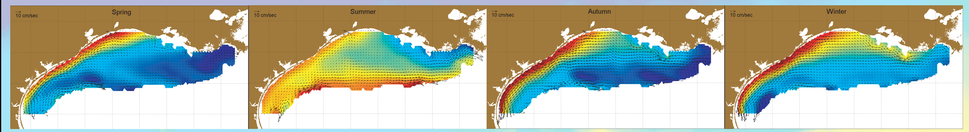
LATEX

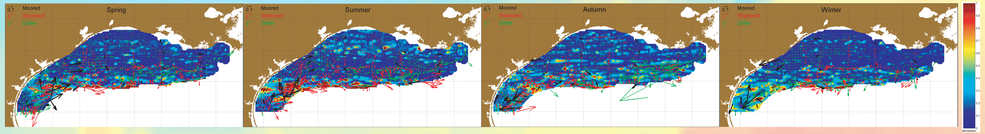


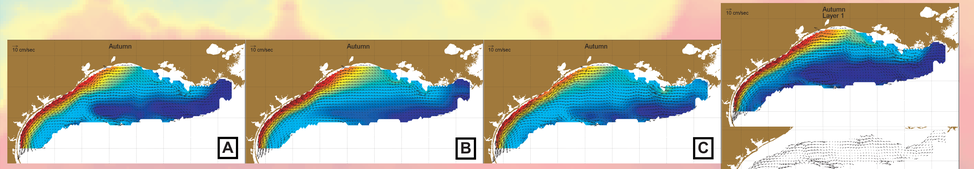


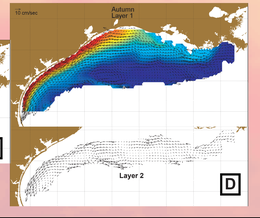












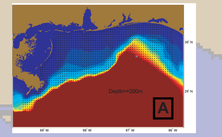
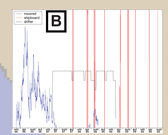
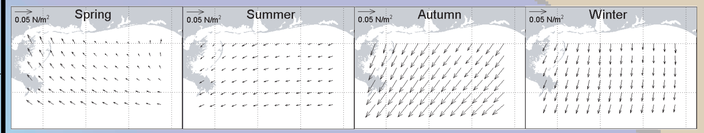
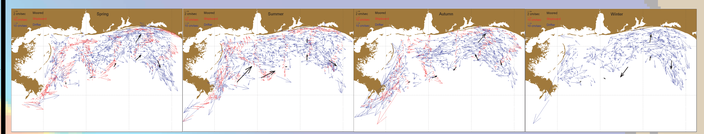
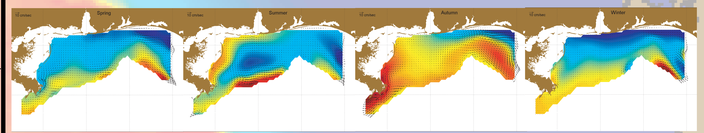
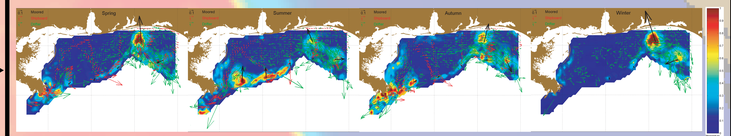
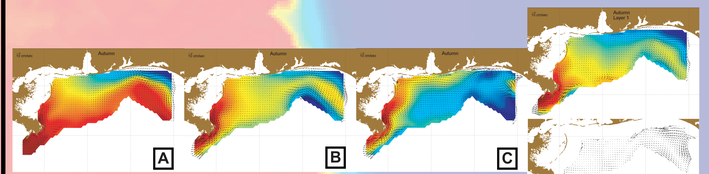
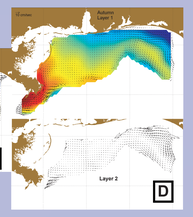
A) Bathymetry of the LATEX shelf and the NEGOM region. Black dots signify the location of grid points at which solutions are computed and the gray stars show the locations of moorings.
B) Time series of the number of measurements for the three different types of velocity measurements (moorings, shipboard ADCP, and drifter) that are assimilated.

Seasonally averaged wind stress fields taken from 5 years of Navy Global Ocean and Atmospheric Prediction System (NOGAPS) data.

Seasonally and depth averaged measurements for spring, summer, autumn, and winter. For the LATEX region, mooring and shipboard ADCP measurements are from the LATEX A study. As for the NEGOM region, these measurements are from the DeSoto Canyon Eddy Intrusion and Northeastern Gulf of Mexico chemical oceanography and hydrography studies. The drifter measurements are from the SCULP drifter data sets.

Spring, summer, autumn, and winter solutions resulting from the assimilation of weighted dynamics with weighted mooring, shipboard ADCP, and drifter data. The vectors represent the seasonally and depth averaged circulation and the colored contours represent SSH.

Residuals of momentum (colored contours), mooring (black arrows), shipboard ADCP (red arrows), and drifter (green arrows) measurements relative to their corresponding expected errors.

Autumn solutions resulting from the assimilation of weighted dynamics with weighted **A)** mooring and shipboard ADCP data, **B)** drifter data, and **C)** mooring, shipboard ADCP, drifter, and TOPEX data.
D) Two layered solutions with an interface depth at 30m, and the inclusion of mooring, shipboard ADCP, and drifter data.

NEGOM