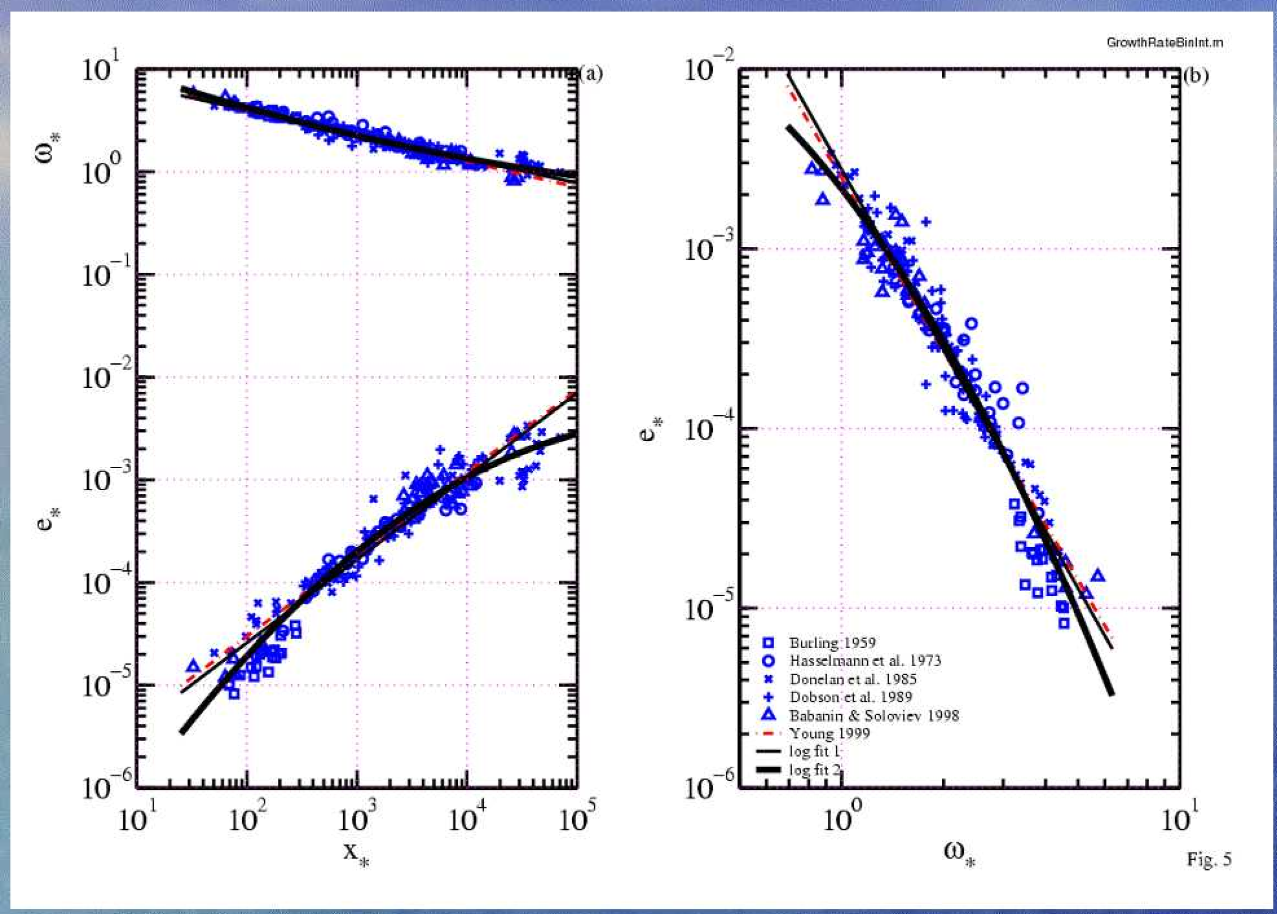
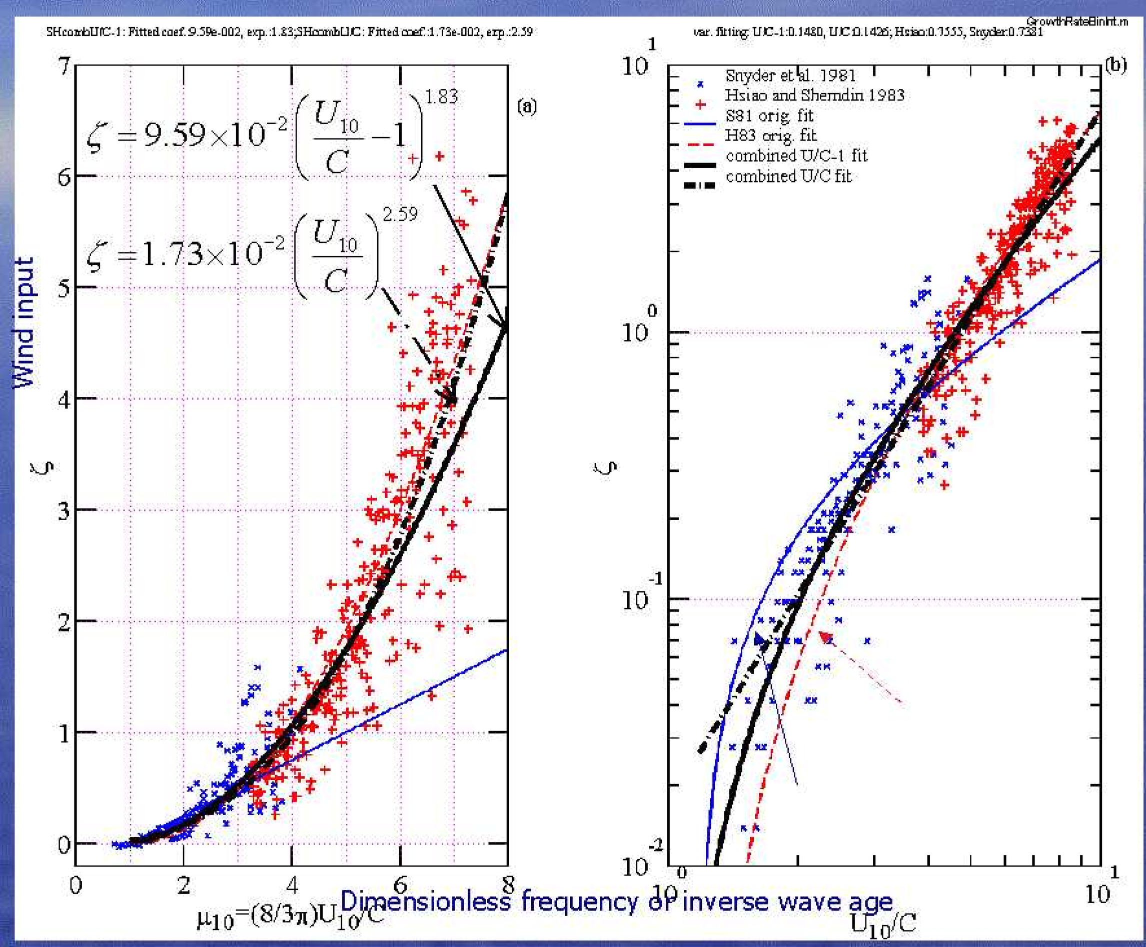
Retention rate of air-sea momentum exchange in the wind-generated ocean wave system

Paul A. Hwang and Erick Rogers

Using the fetch limited growth laws the analytical expression of the net growth rate of wind-generated waves can be derived. To the first order, simple power-law functions yield a sufficient representation of fetch limited wave growth but a second order presentation is more accurate especially for very large and very small dimensionless fetches. The quantitative results of the net growth rates based on first and second order fetch laws are presented. A comparison of the net growth rate with the wind input function is carried out. Non-local balance (in the wavenumber or frequency domain) of the source functions is clearly illustrated, especially for wave components traveling close to the wind speed. In the higher frequency range, the difference between the wind input function and the net growth rate represents a lower bound of the dissipation rate of the wind wave system. The frequency dependence of the derived dissipation function is in good agreement with numerical experiments reported in the literature.

The fetche limited growth functions contain a considerable amount of information on the dynamics of wind-wave generation …





The wind input function

$$C_{gx}\frac{\partial E}{\partial x}=\gamma E$$

$$R\,\omega_*^{-1}\frac{\partial e_*}{\partial x_*}=\gamma_* e_*$$

For example, they can be used to investigate the growth rate and source/sink functions…

$$\gamma_*=\frac{R}{e_*\omega_*}\frac{\partial e_*}{\partial x_*}$$

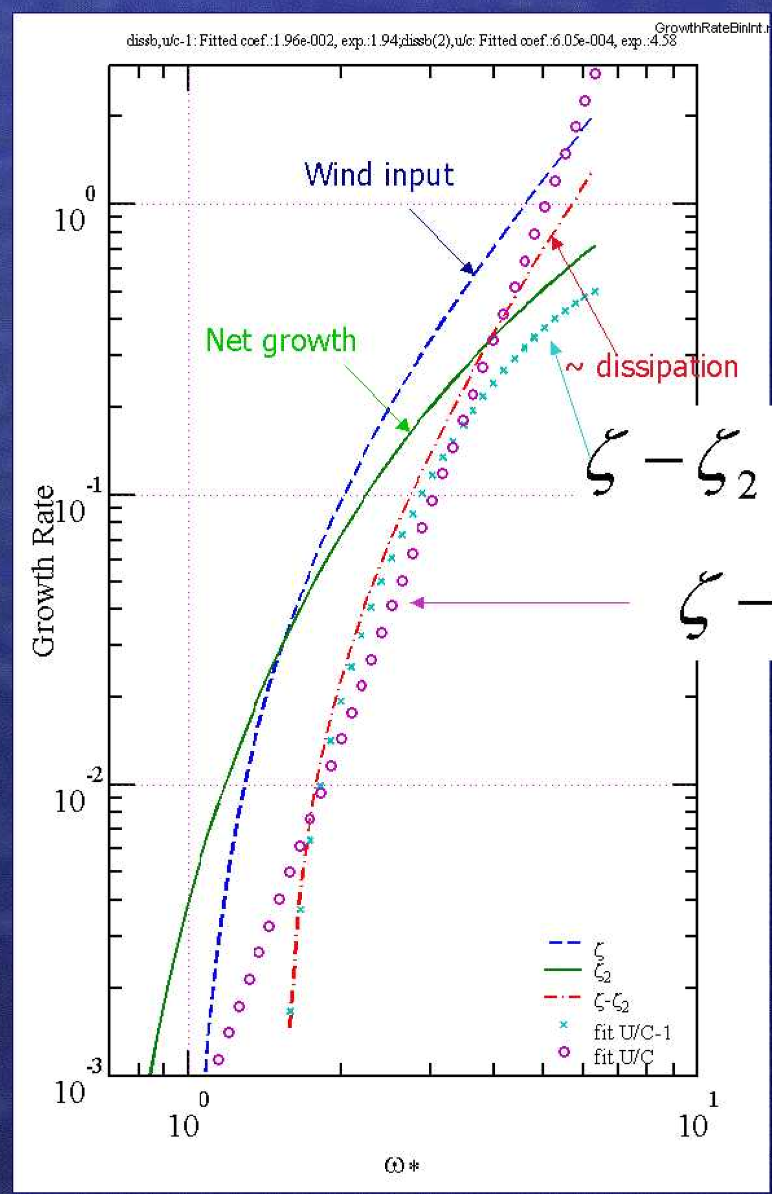
$e_*=\eta_{rms}^2g^2/U_{10}^4$, $x_*=xg/U_{10}^2$, $\omega_*=\omega_pU_{10}/g$, $\gamma_*=\gamma U_{10}/g$, and $R=C_{gx}\omega_p/g$. $R=f[S(f), D(\theta)]$

$$e_*=Ax_*^{a};\quad \omega_*=Bx_*^{b}$$

$$\gamma_{*1}=Rax_*^{-1}\omega_*^{-1}=RaB^{\frac{1}{b}}\omega_*^{\frac{1+b}{-b}}$$

$$\zeta_1=\frac{1}{s}\frac{\gamma_1}{\omega_p}=\frac{1}{s}\frac{\gamma_{*1}}{\omega_*}=\frac{1}{s}Ra\omega_*^{-2}x_*^{-1}=\frac{1}{s}RaB^{\frac{1}{b}}\omega_*^{\frac{1+2b}{-b}}=A_{\zeta\omega}\omega_*^{a_{\zeta\omega}}$$

$$\ln e_*=\sum_{n=0}^{N}a^n(\ln x_*)^n;\ \ln\omega_*=\sum_{n=0}^{N}b_n(\ln x_*)^n$$

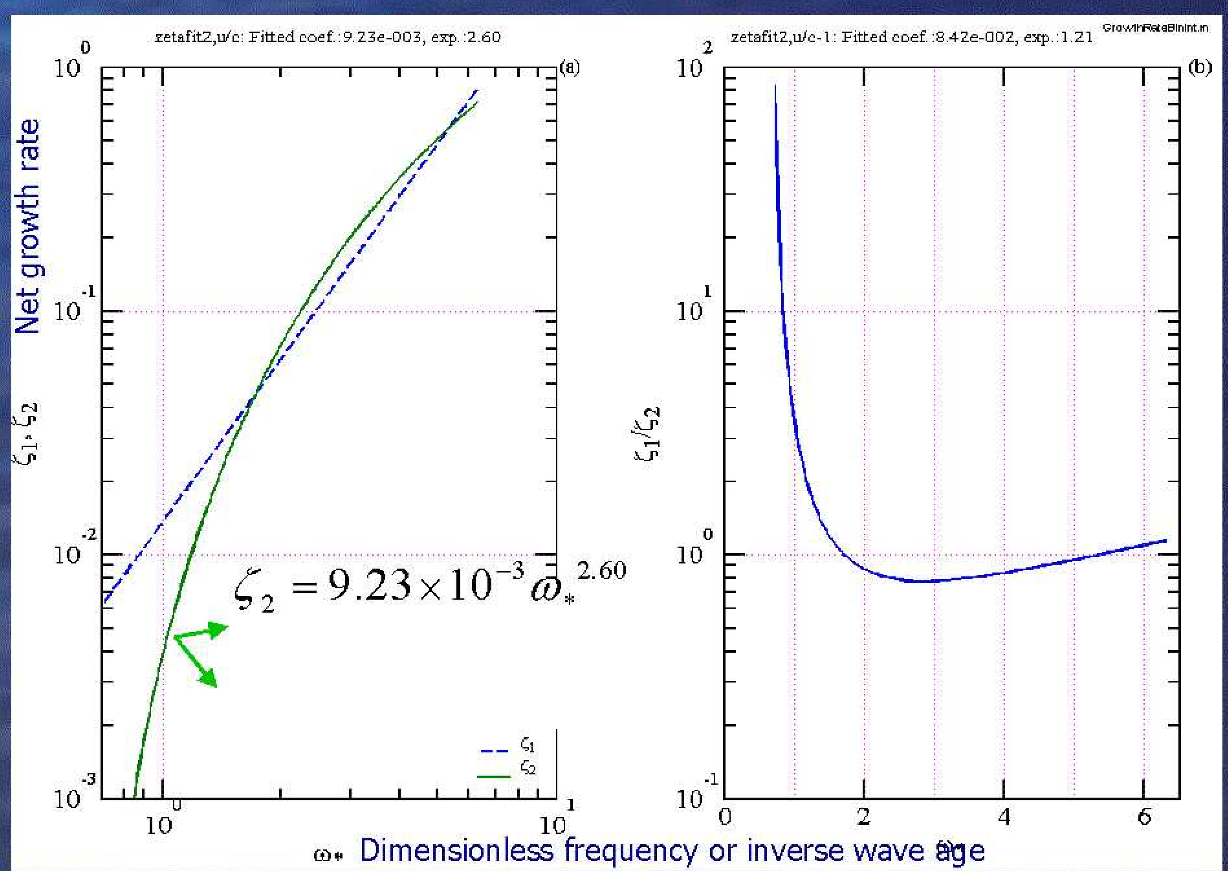


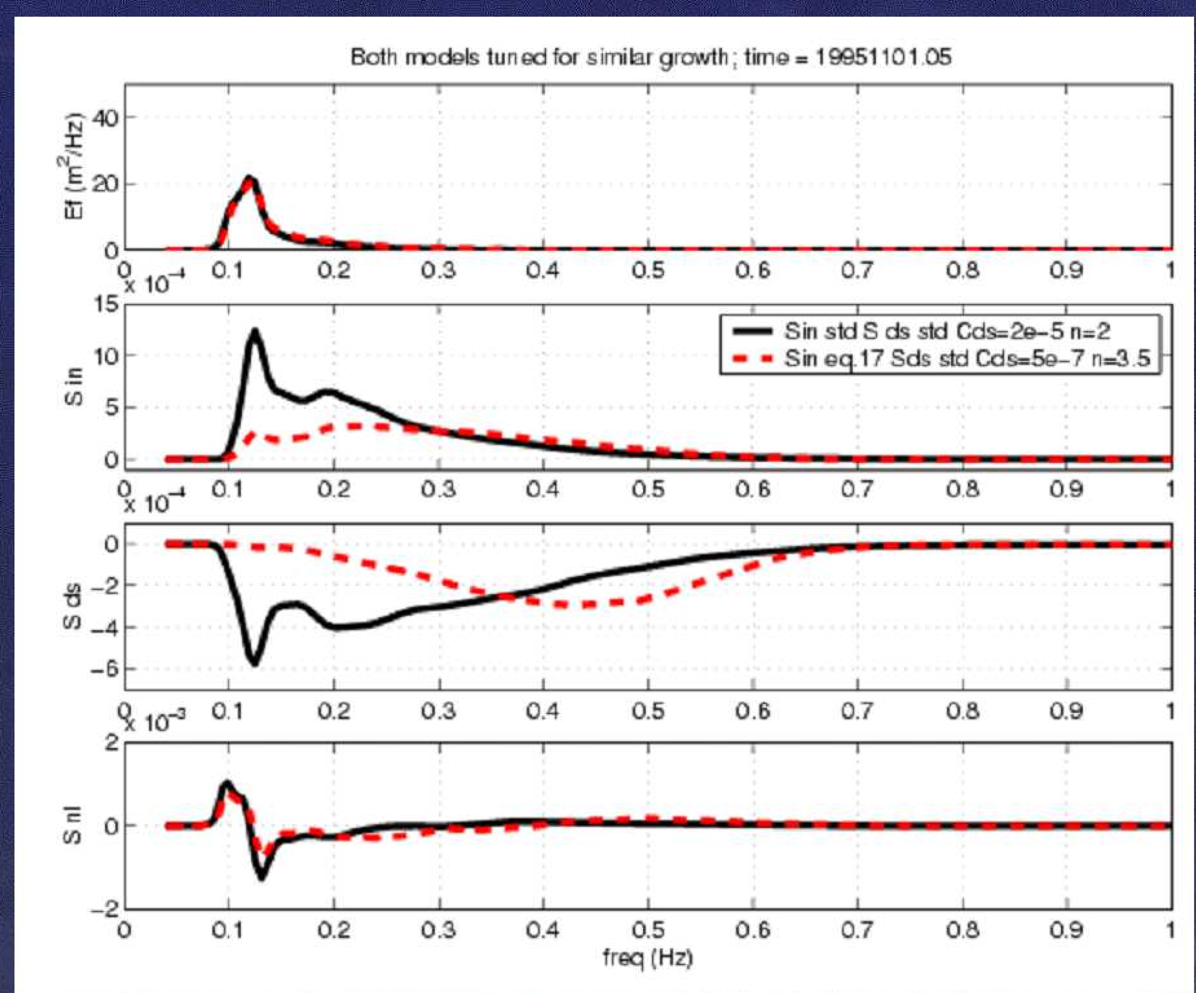
$$\zeta-\zeta_2=1.96\times10^{-2}(\omega_*-1)^{1.94}$$

$$\zeta-\zeta_2=6.05\times10^{-4}\omega_*^{4.58}$$

The dissipation function

The net growth rate





Implications:
Model source functions
Validation and verifications

Work supported by the Office of Naval Research [NRL PE61153N, 62435N]