Statistics of Small-Scale Ocean Wave Groups

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Abstract- **Statistics of small-scale (short capillary-gravity) ocean wave groups are studied with *in-situ* data and group-induced long wave theory. The analyzed data show a close relationship between mean length of runs, average time duration and significant wave slope of a random wave field. They both tend to decrease as wave slopes increase. Group statistics can be described by group-induced long waves and agree with data well.**

I.INTRODUCTION

Wind-generated short capillary and capillary-gravity waves play an important role in ocean remote sensing and air-sea interaction processes. Small-scale ocean wind waves contribute primarily to sea surface roughness elements and affect radar Bragg scattering and air-sea transfer of momentum, mass, energy, and heat. Studies on short wave group structure are few due to the difficulties to measure short waves in the field, even though wave group structure for longer waves (swells) has been studied extensively in the past several decades [2,3,6,8,12].

In-situ measurement of short wind waves requires special technology because short wave signals can be easily contaminated by surface drifts [1], background long waves [9] and interference from mounting structures [4]. Employing a scanning slope sensing buoy on a free-drifting mode to minimize flow disturbance, Hwang et al. [4] measured short wind waves to study wave number spectrum in open ocean environment. The innovated ocean instrument technology was recently up-graded by Hwang at the Naval Research Laboratory (NRL), Stennis Space Center (SSC), Mississippi, adding two wave gauge arrays and several environmental sensors to a new buoy with slope scanning sensor (Fig. 1). The new system is capable of measuring small-scale surface wind waves both temporarily and spatially to resolve wavelengths from as small as 4×10^{-3} to 1 m, covering the band of Bragg resonance waves of most microwave radars used in ocean remote sensing. The system is also able to record wind speeds at two levels, air and water temperatures, surface currents, platform location, and buoy motion.

In this paper, small-scale wave group structure is studied both from *in-situ* data and from the group-induced long wave theory developed by Longuet-Higgins and Stewart [7]. The datasets were collected with NRL/SSC slope scanning sensor and wave gauge array buoy system in the Gulf of Mexico in Oct. 2001. This study focuses on group structure of short capillary-gravity waves with wavelength in the order of 1 m. Statistical properties of small-scale wave groups are presented. The purpose of this study is to better our understanding on wave group formation and evolution at an early development stage of short wind waves. Understanding the small-scale wave group structure helps us to understand wind wave generation and transfer of air-sea fluxes. The *in-situ* data used in this study were measured with advanced oceanographic instrumentation and unique in such kinds of short wave measurement. Through this study, it is found that small wave group statistics have a clear dependence on the significant wave slope of a random wave field. The mean length of runs and average time duration tend to decrease as wave slopes increase. The group-induced long waves can be used to describe short wave group statistics and there is a good agreement between theoretical results and data.

II. INSTRUMENT SYSTEM AND EXPERIMENT

Figure 1 shows the scanning slope sensor and wave gauge array buoy system. The system is constructed on a free-floating platform, supported by subsurface floats. It is deployed in a free-drifting mode to reduce flow disturbance. Hwang et al. [4] described its designation and measuring technique of the instrument system. Only the relevant information is summarized below.

The scanning slope sensor uses optical technique to measure extremely short waves with wavelength ranging from 4×10^{-3} to 0.1 m. A laser beam scans a two dimensional pattern of eight linear segments spaced at 5×10^{-3} m. Each of the linear segments is 0.1 m long, with 50 sampling positions equally spaced at every 2×10^{-3} m. The sampling rate is 50 Hz. The optical scanning speed is 40 m s^{-1}, which is much faster than the phase speed of the measured short waves.

Unlike the scanning slope sensor, the wave gauge array uses electronic thin wires to measure the surface displacement for relatively longer waves with wavelength ranging from 0.1 to 1 m. The array is composed of two sets of capacitance based wave height sensors. Each array has twenty thin wires. The wires are 5×10^{-2} m apart. The axis of each array is orthogonal to the other. The sampling rate is 25 Hz and the resolution of the surface displacement measurement is 2.5×10^{-4} m. The wave gauge array can be operated either independently with the scanning slope sensor to measure short gravity waves (0.1 to 1 m) or they

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Fig. 1. The scanning slope sensor and wave gauge array buoy system recently developed by Hwang at NRL/SSC is designed to measure small scale ocean surface wind waves to resolve wavelengths from as small as 4×10^{-3} to 1 m. The system also records wind speeds, air, water temperatures, humidity, surface currents, swells, and buoy motion.

can be integrated into a whole system to measure short capillary and capillary-gravity waves (4×10^{-3} to 1 m).

Short wind waves, which usually ride on background long waves or surface currents, will be significantly modified [9]. The heights of longer waves are usually several orders of magnitude larger than those of short waves, thus, both sensor systems need to be carried on a wave-following platform to compensate for the motion of long waves and surface currents. Furthermore, the system operates in a free-drifting mode such that the relative velocity between the buoy and surface currents is reduced to a minimum [4]. On top of the platform, a wind vane helps the platform to align with the mean wind direction (Fig. 1). A Sontek Acoustic Doppler Velocimeter (ADV) under sea surface is used to record water velocity component, u, v and w. It samples at a rate of 1 Hz. The environmental sensors record wind speeds and directions at levels of 0.34 and 1 m above the water surface, air and water temperatures, humidity, and location of the buoy. The buoy motion and orientation are also recorded with dynamic sensors for two-axis tilt and three-axis acceleration. Three pressure sensors are used to obtain the water depth and surface slope. These parameters are used for the background long wave measurement and buoy motion correction.

After the scanning slope sensor and wave gauge array buoy system was carefully calibrated in the laboratory, it was used to measure short waves in the Gulf of Mexico in Oct. 2001. The buoy was deployed in different sea conditions: pure wind seas, swells, wind seas mixing with swells, and wind seas in fetch-limited conditions. Ninety-nine datasets were collected. The sea surface displacement data used in this study were from the wave gauge array measurement. The wave group statistics are analyzed for short wave data collected in a fetch-limited condition and described in the following section.

III. DATA ANALYSIS AND RESULTS

The sea surface elevations recorded by the forty wave gauges were analyzed for wave group statistics. Datasets collected in presence of swells and swells with mixing seas conditions were not used because the modulation of short waves by long waves was significant and buoy motion correction become important in both cases. Only datasets obtained in fetch-limited pure sea condition were used in this study. Two datasets were used for wave group statistics. Both were measured in entirely fetch-limited condition in the St. Andrew Bay, Gulf of Mexico. The first data set was measured from 15:37 to 16:17 PM CST and the second one was measured 13 minutes later from 16:30 to 17:10 PM CST on Oct. 12, 2001. Each dataset was recorded 40 minutes long, including a total of 60,000 data points. They were divided into seven 5.46 minutes long data segments with each containing 8,192 data points. The measured wind speeds at 1 m above the water surface were constant and averaged 6 m s^{-1} for the first data set and 6.5 m s^{-1} for the second one. The measured wind directions were steady, aligned with the buoy direction. Each data segment was analyzed with Fast Fourier Transform (FFT) to get

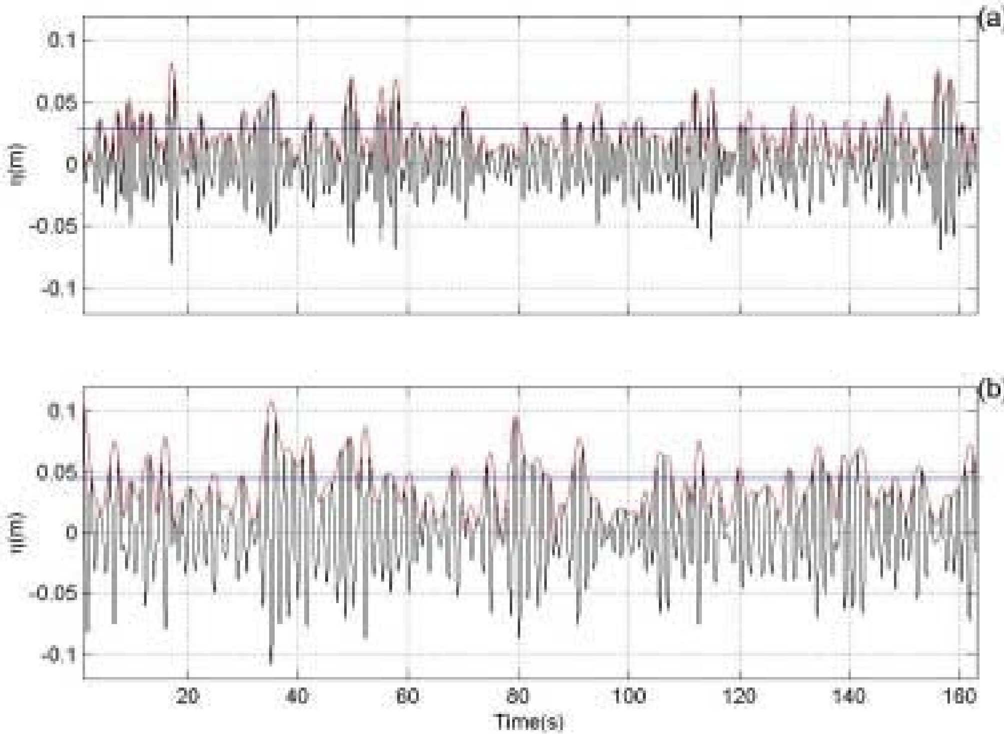

Fig. 2. The filtered time series sea surface elevations at the central wave gauge for the first dataset after a bass pass filter with upper and lower cutoffs at $1.5fp$ and $0.5fp$. (2a) was recorded at the beginning of the deployment. (2b) was recorded 30 minutes later. The wave envelope was calculated from Hilbert Transform. A given level for $\rho_0 = \frac{1}{2} H_{mo}$, is also shown.

frequency wave spectra. The raw spectrum is smoothed with a rectangular window with 24 points.

The unprocessed sea surface elevation data were noisy due to the extremely short waves with wavelength less than 0.4 m. However, these extremely short waves do not contribute to the structure of wave groups and a band pass filter suggested by Longuet-Higgins [8] is thus used to band pass the datasets. The filter has upper and lower cutoffs at $1.5f_p$ and $0.5f_p$, ignoring harmonic components that are either very short or very long compared with the peak wave frequency. The filtered sea surface elevation showed that small wind waves tend to group together. An example of the filtered time series sea surface elevation is given in Fig. 2 for the first dataset. Fig. 2a shows the sea surface elevation at the beginning of the deployment, and Fig. 2b shows the sea surface elevation 30 minutes later. They both showed a good group structure. The mean wave height, H_{mo}, is defined by $\sqrt{2\pi m_0}$, where m_0 is the zeroth moment of the wave spectrum, $m_0 = \int S(\omega) d\omega$ and $\omega = 2\pi f$. A given level, $\rho_0 = \frac{1}{2} H_{mo}$, is also shown in Fig. 2. The wave envelope is calculated from the Hilbert Transform.

According to Goda [3], the mean length of runs, $\overline{l(\rho_0)}$, is defined by the average number of waves exceeding a certain level, ρ_0. The average time duration, $\overline{\tau(\rho_0)}$, is defined by the average time of waves exceeding the level, ρ_0. Corresponding to $\rho_0 = \frac{1}{2} H_{mo}$, both parameters were calculated. The difference in the mean length of runs and average time duration among forty wave gauges is not significant, implying that small wave group statistics do not vary significantly in space. Thus, data from the ten central wave gauges were used for further analysis.

Huang et al. [5] identified a significant wave slope as an important parameter in the wind wave studies. The significant wave slope is defined as $S = \overline{\left(\zeta^2\right)}^{1/2} / L_p$, where $\overline{\left(\zeta^2\right)}^{1/2}$ is the mean square surface elevation and L_p is wavelength at spectral peak. In order to show the relationship between group statistics and significant wave slope, the mean length of runs and average time duration calculated from both datasets are plotted against their significant wave slopes in Fig. 3a-b. The error bars represent the standard deviation from the ten central wave gauges. It is clear that both mean length of runs and time duration are closely related to the significant wave slope of

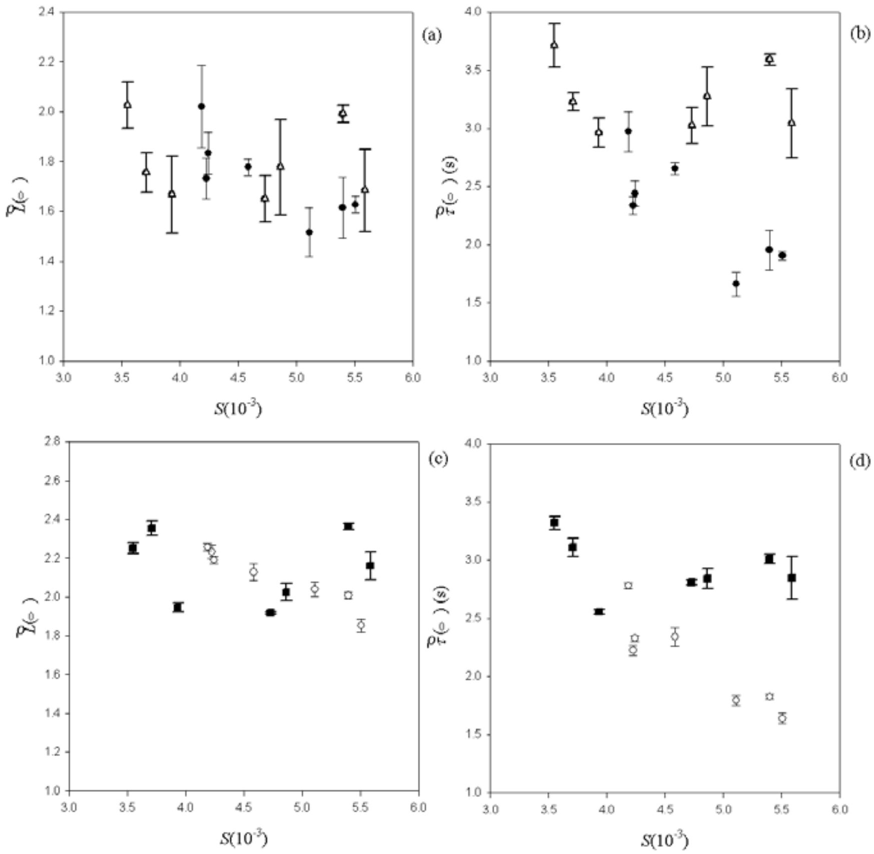

Fig. 3. The mean length of runs, average time duration as a function of significant wave slope. The solid circles in (3a,b) show data points of the first dataset, and the open triangles show data points for the second dataset. The open circles in (3c,d) show results from group-induced long wave method for the first dataset, and the solid squares show results from long wave method for the second dataset. Error bars are the standard deviation from the ten central wave gauges.

a random wave field. They decrease with increasing wave slope.

Lin and Huang [6] derived a parabolic relationship between group-induced long wave and the wave envelope, and used it to represent the group statistics. In theory, this parabolic relationship should apply to the small-scale wave groups. In the following section, the group-induced long wave theory [7] is going to be used to describe the statistics of small wave groups. The long wave theory and its application to the small wave groups in deep water are presented first, followed by an application.

IV. GROUP STATISTICS DESCRIBED BY GROUP-INDUCED LONG WAVES

When a sequence of high waves grouping together, a second order long wave is generated. The mechanism is explained by the concept of radiation stress, which is defined by Longuet-Higgins and Stewart [7] as the excess flow of momentum due to the presence of waves. The long wave theory was extensively applied to discuss near-shore processes such as wave “set-up” and “surf-beat”. Bowers [2] applied it to study harbor resonance due to set-down beneath wave groups. Ottesen Hansen [10,11] derived the long wave elevation from the momentum equation for irregular random waves in shallow water. Sand [12] defined a group factor based on bounded long waves in time domain. Lin and Huang [6] found that the bounded long wave is parabolically related to wave envelope and thus can be used to represent statistical properties of wave groups. The application of the long wave theory is mainly used for waves in shallow water before, however, it is shown in this study that it can also be applied in deep water to depict the statistics of small-scale ocean wave groups.

A. Group-induced long waves in deep water

When a group of waves travel in deep water, variations in the radiation stress cause convergences in the upper ocean layer. Continuity is preserved by pushing water up and down, thus producing variation of water surface. A long period wave is generated by this mechanism [7]. The thickness of the layer, D, is proportional to k^{-1}, where k is the wave number of dominant waves. Since radiation stress is concentrated in this layer, this depth can be called "radiation stress depth". It is defined to be equal to the wavelength of the dominant waves. For a regular wave group, Longuet-Higgins and Stewart [7] showed that the long wave elevation, ξ, is written as

$$\xi = -\frac{\left(a^2 - a_0^2\right)\Delta k}{4\left\{\tanh(h\Delta k) - \Delta k / k\right\}}, \tag{1}$$

where a_0 is a constant and assume to equal to zero for simplicity, a is local wave amplitude, h is water depth and k is wave number of the wave components, and Δk is the difference between the wave numbers. It can be seen from (1) that long wave elevation is directly proportional to the square of local wave heights. The water level is depressed under the largest waves and the long wave is out of phase with wave envelope.

B. Calculation of long waves in deep water

For waves in shallow water, Ottesen Hansen [10] showed that for a simple wave group composed of two Fourier wave components, the long wave elevation $\xi_{nm}(x,t)$ is

$$\xi_{nm}(x,t) = G_{nm}(f_n, f_m) \times [(a_n a_m + b_n b_m)\cos(\Delta\omega_{nm} t - \Delta k_{nm} x) + (a_m b_n - a_n b_m)\sin(\Delta\omega_{nm} t - \Delta k_{nm} x)], \tag{2}$$

where a_n, b_n and a_m, b_m are the Fourier coefficients of the two wave components with frequencies f_n, f_m and wave number k_n, k_m. $\Delta f_{nm} = f_n - f_m$ is frequency and $\Delta k_{nm} = k_n - k_m$ is wave number of the long wave, $G_{nm}(f_n, f_m)$ is a transfer function as function of water depth, h, given by [10,11]. For waves in deep water, the water depth no longer plays a role in ocean wave dynamics. The momentum and continuity equation will be integrated over the radiation stress depth, instead of the whole water column, resulting in a transfer coefficient as a function of radiation stress depth:

$$G_{nm}(f_n, f_m) = \frac{\left[\frac{g}{2}\Delta\omega_{nm}\Delta k_{nm}\left(\frac{1}{c_n} + \frac{1}{c_m}\right)\Delta k_{nm} D \coth(\Delta k_{nm} D) - \frac{1}{2}\Delta k^2{}_{nm}\Delta\omega^2{}_{nm} D + \frac{\omega_n \omega_m \Delta k^2{}_{nm} D \cosh(\Delta k_{nm} D)}{\cosh(\sigma k_{nm} D) - \cosh(\Delta k_{nm} D)}\right]}{\left[\Delta k_{nm} D \coth(\Delta k_{nm} D) - g D \Delta k^2{}_{nm}\right]}, \tag{3}$$

where $\Delta\omega_{nm} = 2\pi\,\Delta f_{nm}$, $\sigma k_{nm} = k_n + k_m$, c_n and c_m are wave phase speed of the two wave components, g is gravity. For an irregular wave group, $\xi(x,t)$ can be expressed as

$$\xi(x,t) = \sum_{n-m=1}^{N} \sum_{m=m^*}^{N} \xi_{nm}(x,t). \tag{4}$$

Where $m^* = f^*/\Delta f$, and f^* is the lowest cutoff frequency in the short wave spectrum, Δf is the frequency interval, and N is the number of wave components.

C. Group statistics described by long waves

The group-induced long waves travel with group velocity. Their amplitudes are directly proportional to the square of local wave heights as shown in (2). For longer waves, group-induced long waves have been observed both in the field [12] and in the laboratory [11]. Lin and Huang [6] found that the group-induced long waves can be parabolically related to the wave envelope, $\rho(x,t)$, such that,

$$\xi(x,t) \approx A\rho^2(x,t) + B, \tag{5a}$$

or equivalently

$$\rho(x,t) \approx \pm\sqrt{\frac{\xi(x,t) - B}{A}}, \tag{5b}$$

for $\xi(x) - B \geq 0$, where

$$A = \frac{1}{2}G(f_P, \Delta f), \qquad B = -\frac{1}{2}G(f_P, \Delta f)m_0. \tag{6}$$

$G(f_p, \Delta f)$ is transfer function at the peak frequency and is estimated by (3).

It is certain that the parabolic relationship between group-induced long waves and the wave envelope in (5a) or (5b) applies to small-scale wave groups with the water depth being replaced by the radiation stress depth in the transfer function. Therefore, the mean length of runs, $\overline{l(\rho_0)}$, for a given level, ρ_0, can be expressed as

$$\overline{l(\rho_0)} = \frac{m_{0\xi}}{m_{1\xi}} \frac{m_1}{m_0} \exp\left(-\frac{\rho_0{}^2}{2m_0}\right), \tag{7}$$

where $m_{0\xi}$ and $m_{1\xi}$ are the zeroth and first moment of long wave spectrum. The average time duration, $\overline{\tau(\rho_0)}$, for a given level, ρ_0, can be written as

$$\overline{\tau(\rho_0)} = 2\pi \frac{m_{0\xi}}{m_{1\xi}} \exp\left(-\frac{\rho_0{}^2}{2m_0}\right). \tag{8}$$

Specifically, if the given level ρ_0 is equal to one half of the mean wave height,

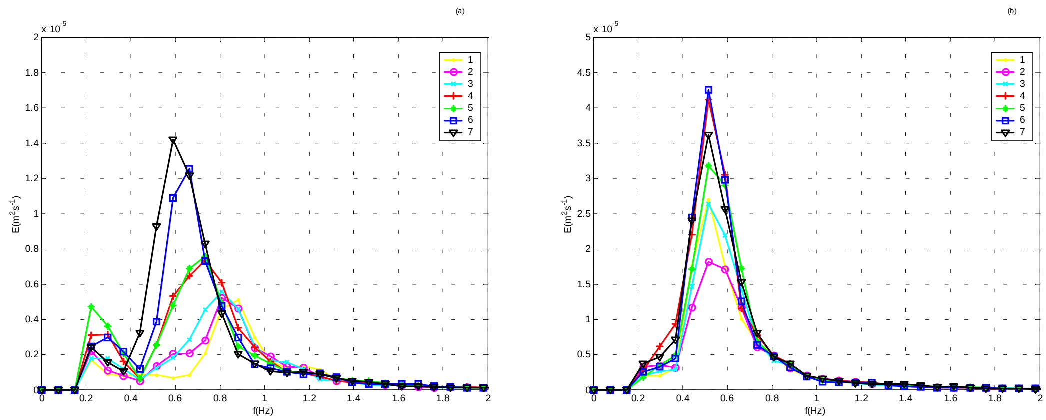

Fig.4. The seven successive wave energy spectra for the first dataset (4a) and the second dataset (4b). Different lines show wave spectra at different time with 5.46 minutes (8192 points) intervals. The peak energy and frequency change with time, resulting in a different significant wave slope.

$$\rho_0 = \tfrac{1}{2} H_{mo} = \sqrt{\frac{\pi}{2} m_0}\,, \tag{9}$$

the mean length of runs and average time duration can be expressed as

$$\overline{l(\tfrac{1}{2} H_{mo})} = \frac{m_{0\xi}}{m_{1\xi}} \frac{m_1}{m_0} \exp\left(-\frac{\pi}{4}\right), \tag{10}$$

$$\overline{\tau(\tfrac{1}{2} H_{mo})} = 2\pi \frac{m_{0\xi}}{m_{1\xi}} \exp\left(-\frac{\pi}{4}\right) \tag{11}$$

D. Application of the long wave method to small wave groups

After the band pass filter was applied to each data segments of the sea surface elevation, group-induced long waves were calculated with (2) – (4). The mean length of runs and average time duration were calculated with (10) and (11) for the sea surface elevations from the ten central wave gauges. Their mean values with standard deviation are plotted in Fig. 3c,d against the significant wave slopes. Results calculated from the long wave method agree with the data well with a significant smaller standard deviation. Furthermore, the pattern of variation of the mean length of runs and average time duration as a function of significant wave slope shows an excellent agreement with the data. It is clear that both the mean length of runs and average time duration depend upon the wave slope. With wave slopes increase, the mean length of runs and average time duration tend to decrease, as shown in Fig. 3.

V. SUMMARY AND DISCUSSION

The close relationship between group statistics and the significant wave slope indicates that the evolution of wave energy spectra play a critical role in determining the wave group statistics because the wave slope is closely related to the spectral energy level [5]. The seven consecutive frequency wave energy spectra calculated from the central wave gauge for the first dataset are plotted in Fig. 4a. At the very beginning of the short wind wave development, the short wave spectrum has a peak frequency, f_p, of 0.882 Hz and the peak energy, E_{max}, of 5.07×10^{-6} m^2/s. The next spectrum shows that 5.46 minutes later, f_p decreased to 0.809 Hz and E_{max} increased to 5.20 $\times10^{-6}$ m^2/s. The increase of wave energy and the decrease of peak frequency with time is the dominated characteristic of the evolution of the short wind wave spectra. Fig. 4b is the seven successive wave spectra for the second data set. It is very clear that all the short wave spectra in Fig. 4b have a same peak frequency, f_p=0.516 Hz (T_p=1.94 s). The peak wave energy changed with time from 27×10^{-6} m^2/s to 18.1×10^{-6}, 26.3×10^{-6}, 41.1×10^{-6} m^2/s. But when the wind wave growth is limited by fetch, the wave energy no longer increase, as well as the peak frequency.

The mean wave height and peak frequency change with the evolution of the short wind wave spectra, which is shown in Fig. 4. As a result, the significant wave slope will vary with the wave spectral evolution. The mean length of runs and average time duration increase with time at the early development of the short wind waves, due to the decrease of the significant wave slope. With the increment of wave spectral energy, the mean length of runs and average time duration increase as well. This indicates that significant wave slope is an important parameter not only for determining the statistical properties of the random surface [5], but also for determining the statistical properties of wave groups.

In a summary, small-scale wave group statistics are investigated with both *in-situ* data and group-induced long waves. The data were collected with NRL/SSC scanning slope sensor and wave gauge array buoy system. Two datasets obtained in a purely fetch-limited wind wave growth condition were used to analyze the relationship between the mean length of runs, average time duration and the significant wave slope. Results show that the mean length of runs and average time duration depend on wave slope. They both decrease as wave slope increases. The group-induced long waves can be used to describe the small-scale wave group statistics. Results from the long wave method agree with the data well and show a same relationship between the group statistics and wave slope.

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