Light Scattering Phase Functions Measured in Waters of Mediterranean Sea

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Abstract — In this paper we propose 32 new experimental phase functions measured *in situ* during several oceanographic expeditions in various regions of Mediterranean Sea. As in the case with the Petzold phase functions [1] these measurements include a complete set of inherent optical properties for the wavelength of light 520 nm. In 30 cases the beam attenuation coefficient was also measured. The backscattering coefficient, probability of backscattering, and Gordon's parameter are calculated. The complete results are presented in the form of tables for 33 scattering angles in the range of 2 and 162.5 degrees.

1. INTRODUCTION

Light scattering phase functions by seawater that are experimentally measured over thirty years by Petzold became very popular among scientists involving in radiative transfer modeling in the ocean. The original work of Petzold is published as a report [1] and gives numerical values of fifteen phase functions with the values of absorption, scattering and backscattering coefficients. Since the release of Petzold report there were very few publications [2-4] devoted to the experimental seawater light scattering phase functions.

In this paper we propose additional new experimental phase functions measured during several oceanographic expeditions in various regions of Mediterranean Sea and compare their major characteristics with the properties of Petzold phase functions.

For the remote optical sensing of oceans and inland water basins and estimation of visibility properties of seawater it is very important to know relationships between some inherent optical properties. The practically useful relationship between these properties based on 32 new and 15 Petzold phase functions is proposed.

2. MEASURING DEVICES

The light source in the angular nephelometer, used for measuring the angular scattering coefficients, emitted non-polarized light. The detector also was non-sensitive to the polarization of measured light. The full set of technical parameters of the angular nephelometer [5] is given in Table 1.

In situ measurements of presented phase functions were carried out near the sea surface and two depths of 20 and 40 meters. The time of phase function measurement was about 8 minute. In most cases we usually made three measurements and computed an average value. The signal from the submerged nephelometer was transmitted to the laboratory on the board of research vessel through the cable and was recorded to a digital tape.

The beam attenuation coefficient was measured with a separate laboratory nephelometer described in article [6]. That nephelometer has an interference filter centered at 520 nm and has a precision of measurement in the range of 10%.

3. RESULTS OF MEASUREMENTS

The dates, depths and locations of measured 32 phase functions are given in Table 2. The numerical values of the natural logarithms of these phase functions, $\ln[\beta(\vartheta)/b_0]$, at 33 scattering angles ϑ and five basic integral optical properties of water: beam attenuation coefficient c, scattering coefficient b, backscattering coefficient b_B, probability of backscattering $B = b_B/b$, and Gordon's parameter [7, 8]

Table 1. Angular nephelometer specifications [5].

Range of angular measurements	2° - 162.5°
Angular scanning step	5°
Angle of divergence of illuminating beam	1°
Visual angle of detector	1° 7’
Angle reading accuracy	0.2°
Accuracy of measurement of scattering light	12%
Wavelength of maximum sensitivity	520 nm
Half-width of sensitivity band	40 nm.
Maximum depth of submerging	200 m

Table 2. Date, location, and depth of measurements of the Mediterranean light scattering phase functions.

#	# [5]	Date	z,m	Coordinates
01	*12*	10/11/92	0	45° 23.0' N 13° 10.0' E
02	21	05/11/98	0	36° 25.3' N 17° 53.1' E
03	22	05/11/98	0	36° 29.1' N 16° 50.6' E
04	23	05/12/98	0	37° 10.7' N 11° 25.2' E
05	24	05/13/98	0	37° 26.1' N 04° 47.2' E
06	*14*	05/13/98	0	37° 20.1' N 03° 53.8' E
07	25	05/14/98	0	36° 41.9' N 01° 11.1' W
08	26	05/14/98	0	36° 31.8' N 02° 18.2' W
09	*13*	05/15/98	0	35° 59.7' N 05° 26.9' W
10	*17*	05/15/98	0	36° 17.1' N 07° 06.3' W
11	*18*	05/21/98	0	36° 27.4' N 02° 02.2' W
12	*19*	05/21/98	0	36° 42.8' N 00° 26.4' W
13	27	05/22/98	0	37° 19.7' N 06° 23.8' E
14	*15*	05/23/98	0	37° 04.9' N 11° 30.6' E
15	28	05/23/98	0	37° 23.5' N 10° 28.1' E
16	29	05/23/98	0	36° 54.3' N 12° 34.6' E
17	30	05/24/98	0	35° 52.1' N 17° 14.9' E
18	*10*	05/25/98	0	35° 37.0' N 18° 16.0' E
19	31	05/25/98	0	34° 00.0' N 23° 30.0' E
20	32	05/25/98	20	34° 00.0' N 23° 30.0' E
21	33	05/26/98	20	35° 00.0' N 28° 00.0' E
22	34	05/26/98	0	35° 00.0' N 28° 00.0' E
23	*16*	05/27/98	0	35° 15.0' N 29° 15.0' E
24	35	05/27/98	20	35° 15.0' N 29° 15.0' E
25	36	05/27/98	0	36° 00.0' N 29° 00.0' E
26	*20*	05/28/98	0	35° 55.0' N 28° 05.0' E
27	37	05/28/98	40	35° 55.0' N 28° 05.0' E
28	38	05/29/98	0	36° 50.0' N 26° 22.6' E
29	39	05/29/98	0	37° 42.3' N 25° 49.5' E
30	*09*	05/30/98	0	39° 43.0' N 25° 45.0' E
31	40	05/30/98	0	40° 00.0' N 25° 48.9' E
32	41	05/30/98	0	39° 39.0' N 25° 27.0' E

$g = b_B/(a+b_B)$, where $a = c - b$ is an absorption coefficient of sea water, are presented in Tables 3-9. Here $\beta(\vartheta)$ is an angular scattering coefficient in $(\mathrm{m \cdot sr})^{-1}$, and $b_0 = 1\,\mathrm{m}^{-1}$. The numbers in the first column of Table 2 represent a chronological numeration of 32 Mediterranean phase functions. The numbers in the second column represent an older numeration introduced in [3], and numbers in that column represented in *italics* mark the phase functions previously proposed and discussed in [3]. The article [3] presents twenty *in situ* phase functions measured in Mediterranean and Black Seas, Indian, Atlantic and Southern Oceans, and Lake Baykal in Siberia. All eleven Mediterranean phase functions presented in [3] and marked in column 2 by *italics* are included here for completeness. The values of scattering coefficient b were estimated by integrating values of the angular scattering coefficient $\beta(\vartheta)$. To do this, we used a parabolic approximation to smaller angles ϑ using values of $\log[\beta(\vartheta)/b_0]$ against ϑ at 2, 7.5, and 12.5 degrees. The extrapolation to the larger than 162.5 degrees was as follows: $\beta(\vartheta > 162.5°) = \beta(162.5°)$. The values of beam attenuation coefficient c for the two phase functions No. 1 and 23 were not measured. Table 9 also gives an average values of $\ln[\beta(\vartheta)/b_0]$ and 5 basic optical properties for the presented 32 phase functions, with the upper and lower limit values defined by the standard deviation σ from the $\ln[\beta(\vartheta)/b_0]$ and corresponding optical properties.

Table 3. Natural logarithms of Mediterranean phase functions $\ln[\beta(\vartheta)/b_0]$ of light scattering No. 1-5.

ϑ \ PhF #	1	2	3	4	5
2.0	2.0792	1.6413	1.0887	1.3650	1.5723
7.5	-1.3677	-2.3355	-2.3816	-1.8750	-2.2434
12.5	-2.2450	-3.3495	-3.3495	-2.8429	-3.3495
17.5	-2.7907	-4.1950	-4.1029	-3.4812	-4.1950
22.5	-3.7233	-4.8749	-4.9901	-4.2072	-4.8749
27.5	-4.4118	-5.4930	-5.4930	-4.8944	-5.4930
32.5	-4.8815	-5.9402	-5.9402	-5.5487	-5.9402
37.5	-5.3328	-6.4140	-6.3219	-5.9535	-6.4831
42.5	-5.6644	-6.8855	-6.7473	-6.3098	-6.7473
47.5	-6.0604	-7.2125	-7.0744	-6.5218	-7.0744
52.5	-6.3344	-7.3004	-7.3004	-7.2083	-7.3004
57.5	-6.6176	-7.4004	-7.5616	-7.4004	-7.5616
62.5	-6.9354	-7.9026	-7.7415	-7.3500	-7.7415
67.5	-7.1012	-8.0001	-8.0001	-7.4705	-8.0001
72.5	-7.5087	-8.2676	-8.1064	-7.8301	-8.2676
77.5	-7.4834	-8.4054	-8.2442	-7.8067	-8.4054
82.5	-7.6077	-8.3900	-8.3900	-7.9295	-8.3900
87.5	-7.7137	-8.4514	-8.4514	-8.2212	-8.3824
92.5	-7.8748	-8.8314	-8.8314	-8.0945	-8.4860
97.5	-7.9071	-8.5282	-8.5282	-7.9986	-8.5282
102.5	-7.9439	-8.5781	-8.5781	-8.0485	-8.5781
107.5	-7.9923	-8.6475	-8.6475	-8.1180	-8.6475
112.5	-8.0452	-8.7254	-8.6563	-8.1958	-8.7254
117.5	-7.9946	-8.7431	-8.7431	-8.1444	-8.7431
122.5	-7.7459	-8.6899	-8.6899	-8.2524	-8.6899
127.5	-7.8541	-8.6474	-8.6474	-8.2099	-8.6474
132.5	-7.8334	-8.7783	-8.6401	-8.3408	-8.6401
137.5	-7.8518	-8.6584	-8.7966	-8.3821	-8.6584
142.5	-7.7275	-8.6705	-8.6705	-8.1179	-8.6705
147.5	-7.7597	-8.4730	-8.6342	-8.2197	-8.6342
152.5	-7.8656	-8.5555	-8.4634	-8.1180	-8.4634
157.5	-7.7989	-8.4209	-8.4209	-7.7762	-8.4209
162.5	-7.6262	-8.1785	-8.2936	-7.6489	-8.1785
b, 1/m	0.4099	0.0979	0.0644	0.0926	0.0927
c, 1/m	n/a	0.2119	0.1747	0.2061	0.2062
b_B, 1/m	0.0055	0.0011	0.0011	0.0019	0.0012
B	0.0133	0.0116	0.0175	0.0205	0.0127
g	n/a	0.0099	0.0101	0.0165	0.0103

For the purposes of optical remote sensing it is useful to have a reliable relationship between

Table 4. Natural logarithms of Mediterranean phase functions of light scattering $\ln[\beta(\vartheta)/b_0]$ No. 6-10

ϑ \ PhF #	6	7	8	9	10
2.0	1.5262	1.6413	1.5492	1.3420	1.0887
7.5	-1.8750	-2.3355	-2.1513	-2.2895	-2.1974
12.5	-2.9120	-3.2804	-3.4186	-3.4876	-3.3495
17.5	-3.6424	-3.8266	-3.9187	-4.2641	-4.0108
22.5	-4.2993	-4.5296	-4.6217	-4.8059	-4.5296
27.5	-5.0555	-4.8944	-5.2628	-5.4009	-5.0555
32.5	-5.5487	-5.2494	-5.4336	-5.7329	-5.6408
37.5	-5.9535	-5.8153	-5.9535	-6.3219	-5.9535
42.5	-6.3098	-6.0796	-6.3789	-6.6091	-6.4940
47.5	-6.6599	-6.2224	-6.5218	-7.0744	-6.5218
52.5	-7.0011	-6.5866	-6.8629	-7.2083	-7.1392
57.5	-7.2393	-7.0781	-7.1472	-7.4004	-7.1472
62.5	-7.5112	-7.0967	-7.1888	-7.6263	-7.1888
67.5	-7.7007	-7.3093	-7.5856	-7.7007	-7.3093
72.5	-7.8301	-7.4387	-7.8301	-7.9683	-7.6689
77.5	-7.9449	-7.6455	-7.9449	-7.9449	-7.8067
82.5	-8.0676	-7.7913	-7.9295	-8.2288	-7.9295
87.5	-8.2212	-8.0600	-8.0600	-8.2212	-8.0600
92.5	-8.4169	-8.0945	-8.2557	-8.4169	-8.2557
97.5	-8.2979	-7.9986	-8.2979	-8.2979	-8.4591
102.5	-8.5090	-8.0485	-8.1867	-8.5090	-8.5781
107.5	-8.4173	-8.1180	-8.2561	-8.5785	-8.6475
112.5	-8.6563	-8.1958	-8.4951	-8.7254	-8.6563
117.5	-8.5819	-8.1444	-8.2826	-8.5819	-8.5819
122.5	-8.5287	-8.2524	-8.3906	-8.5287	-8.5287
127.5	-8.5093	-8.2099	-8.5093	-8.5093	-8.6474
132.5	-8.5020	-8.2257	-8.3408	-8.7783	-8.6401
137.5	-8.4973	-8.2670	-8.2670	-8.6584	-8.6584
142.5	-8.4402	-8.1179	-8.1179	-8.8317	-8.4402
147.5	-8.4730	-8.2197	-8.3118	-8.7493	-8.3118
152.5	-8.4634	-8.1180	-8.1180	-8.4634	-8.4634
157.5	-8.2828	-7.8913	-7.7762	-8.5591	-8.6972
162.5	-8.1094	-7.6489	-7.7640	-8.2936	-8.2936
b, 1/m	0.2303	0.1052	0.0952	0.1796	0.1589
c, 1/m	0.2671	0.2201	0.2089	0.2072	0.2164
b_B, 1/m	0.0031	0.0019	0.0017	0.0028	0.0029
B	0.0135	0.0182	0.0175	0.0158	0.0182
g	0.0777	0.0164	0.0144	0.0930	0.0478

Table 5. Natural logarithms of Mediterranean phase functions of light scattering $\ln[\beta(\vartheta)/b_0]$ No. 11-15

ϑ \ PhF #	11	12	13	14	15
2.0	0.8354	1.3420	1.5032	0.6282	1.2499
7.5	-2.7730	-1.8750	-1.6678	-3.3487	-2.5658
12.5	-3.7640	-3.4186	-3.0962	-4.0633	-3.7640
17.5	-4.4483	-4.1029	-3.8266	-4.7706	-4.3562
22.5	-4.8749	-4.6907	-4.5296	-5.2434	-4.9901
27.5	-5.4930	-5.4930	-4.8944	-5.7924	-5.4930
32.5	-5.9402	-5.8250	-5.3415	-6.1935	-5.9402
37.5	-6.4831	-6.1837	-5.7002	-6.5982	-6.4140
42.5	-6.6091	-6.4940	-6.2177	-6.8855	-6.7473
47.5	-7.0744	-6.9362	-6.5218	-7.2125	-6.9362
52.5	-7.1392	-7.2083	-6.7248	-7.4616	-7.2083
57.5	-7.2393	-7.1472	-6.9399	-7.6768	-7.2393
62.5	-7.5112	-7.3500	-7.0277	-7.7415	-7.6263
67.5	-7.5856	-7.4705	-7.4705	-8.0001	-7.8619
72.5	-7.8301	-7.6689	-7.5538	-8.2676	-7.9683
77.5	-8.0830	-8.0830	-7.5304	-8.4054	-8.0830
82.5	-8.2288	-8.2288	-7.7913	-8.3900	-8.2288
87.5	-8.3824	-8.3824	-7.7837	-8.4514	-8.3824
92.5	-8.2557	-8.2557	-7.9564	-8.4860	-8.4169
97.5	-8.2979	-8.1367	-7.9986	-8.5282	-8.2979
102.5	-8.1867	-8.5090	-8.0485	-8.5090	-8.1867
107.5	-8.5785	-8.5785	-7.9798	-8.6475	-8.5785
112.5	-8.4951	-8.3339	-8.0576	-8.6563	-8.4951
117.5	-8.2826	-8.4207	-7.9832	-8.7431	-8.4207
122.5	-8.2524	-8.5287	-8.0912	-8.6899	-8.3906
127.5	-8.2099	-8.6474	-8.0948	-8.6474	-8.3711
132.5	-8.5020	-8.5020	-8.2257	-8.6401	-8.5020
137.5	-8.4973	-8.4973	-8.3821	-8.6584	-8.4973
142.5	-8.4402	-8.5554	-8.2791	-8.5554	-8.2791
147.5	-8.4730	-8.6342	-8.3118	-8.6342	-8.3118
152.5	-8.3943	-8.5555	-8.2561	-8.5555	-8.4634
157.5	-7.8913	-8.2828	-8.0065	-8.6972	-8.4209
162.5	-8.1785	-8.1785	-7.8791	-8.4087	-8.1094
b, 1/m	0.1174	0.1911	0.1013	0.0921	0.0697
c, 1/m	0.1566	0.2233	0.2157	0.1151	0.1805
b_B, 1/m	0.0036	0.0032	0.0020	0.0026	0.0014
B	0.0303	0.0168	0.0193	0.0287	0.0208
g	0.0833	0.0903	0.0168	0.1028	0.0129

backscattering and beam scattering coefficients. These relationships based on 15 Petzold phase functions [1] and 32 Mediterranean phase functions, presented in tables 3-9, are shown in Fig. 1 and Fig. 2. Analytically, this relationship may be represented as the following regression:

$$b_B = 0.00076282 + 0.011482\, b + 0.0042513\, b^2, \quad (0 \le b \le 2\,\mathrm{m}^{-1}) \quad r^2 = 0.98944\,. \tag{1}$$

Figure 3 displays an average Mediterranean phase function with the spread determined by the standard deviation σ.

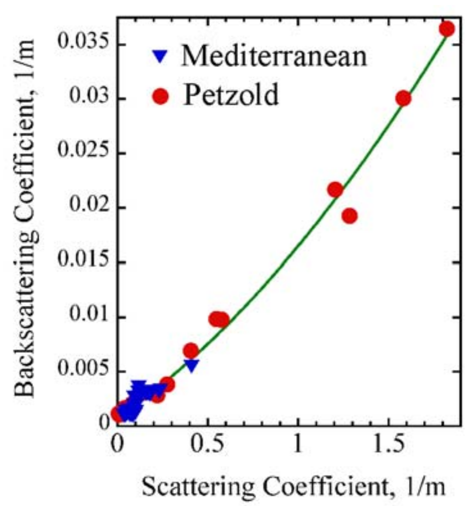

Figure 1. Dependence between backscattering b_B and scattering b coefficients.

Table 6. Natural logarithms of Mediterranean phase functions of light scattering $\ln[\beta(\vartheta)/b_0]$ No. 16-20

ϑ \ PhF #	16	17	18	19	20
2.0	1.5032	1.2959	0.7663	1.6644	1.3190
7.5	-3.1644	-2.8651	-3.2105	-2.8191	-2.9112
12.5	-4.0633	-3.7640	-3.2701	-3.6488	-3.9712
17.5	-4.7706	-4.3562	-4.5402	-4.1950	-4.8627
22.5	-5.3585	-4.9901	-5.1513	-4.8749	-5.3585
27.5	-5.8845	-5.4930	-5.5852	-5.1707	-5.7924
32.5	-6.3086	-5.8250	-6.0784	-5.8250	-6.4467
37.5	-6.5982	-6.4140	-6.4139	-6.4140	-6.5982
42.5	-7.0236	-6.4940	-6.7473	-6.3789	-6.8855
47.5	-7.3737	-7.0744	-7.2126	-6.9362	-7.3737
52.5	-7.6228	-7.2083	-7.4615	-7.2083	-7.6228
57.5	-7.7919	-7.4004	-7.6766	-7.5616	-7.6768
62.5	-8.0408	-7.6263	-7.7415	-7.6263	-7.7415
67.5	-8.1382	-7.7007	-7.8619	-8.0001	-8.2994
72.5	-8.2676	-7.9683	-8.2677	-8.1064	-8.2676
77.5	-8.4745	-8.0830	-8.2442	-8.0830	-8.4054
82.5	-8.4591	-8.2288	-8.4590	-8.3900	-8.4591
87.5	-8.7968	-8.3824	-8.4514	-8.2212	-8.4514
92.5	-8.8314	-8.4169	-8.4859	-8.4169	-8.9925
97.5	-8.8735	-8.2979	-8.5281	-8.5282	-8.5282
102.5	-8.9235	-8.5090	-8.9234	-8.3478	-9.0847
107.5	-8.9929	-8.5785	-8.9930	-8.5785	-8.9929
112.5	-9.0708	-8.6563	-8.7254	-8.6563	-9.0708
117.5	-8.8122	-8.5819	-8.8122	-8.4207	-8.8122
122.5	-8.8511	-8.5287	-8.8509	-8.6899	-8.8511
127.5	-8.8086	-8.5093	-8.8085	-8.6474	-8.8086
132.5	-8.7783	-8.6401	-8.7784	-8.6401	-8.7783
137.5	-8.9348	-8.4973	-8.6584	-8.6584	-8.9348
142.5	-8.8317	-8.4402	-8.6704	-8.4402	-8.5554
147.5	-8.7493	-8.3118	-8.6342	-8.3118	-8.7493
152.5	-8.8778	-8.2561	-8.4634	-8.3943	-8.3943
157.5	-8.7663	-8.0065	-8.2829	-8.1446	-8.2828
162.5	-8.6850	-7.8791	-8.1095	-7.8791	-8.1094
b, 1/m	0.0842	0.0725	0.1151	0.1016	0.0699
c, 1/m	0.1967	0.1837	0.1681	0.2161	0.1808
b_B, 1/m	0.0009	0.0014	0.0026	0.0013	0.0010
B	0.0106	0.0192	0.0222	0.0132	0.0144
g	0.0079	0.0124	0.0461	0.0116	0.0090

Table 7. Natural logarithms of Mediterranean phase functions of light scattering $\ln[\beta(\vartheta)/b_0]$ No. 21-25

ϑ \ PhF #	21	22	23	24	25
2.0	0.9506	1.2959	0.8815	0.7203	0.7203
7.5	-3.0723	-2.9112	-3.2565	-3.4868	-2.8651
12.5	-3.8791	-3.8100	-4.1554	-4.3396	-3.8100
17.5	-4.6555	-4.7706	-4.6555	-5.0469	-4.3562
22.5	-5.2434	-5.2434	-5.3585	-5.4506	-5.0822
27.5	-5.7003	-5.7003	-5.4930	-5.8845	-5.4009
32.5	-6.1935	-6.1935	-6.0783	-6.4467	-6.0783
37.5	-6.5982	-6.5982	-6.4140	-6.4831	-6.4140
42.5	-7.0236	-7.0236	-6.8855	-7.3690	-6.8855
47.5	-7.2816	-7.2816	-7.0744	-7.6961	-7.0744
52.5	-7.4616	-7.6228	-7.3004	-7.8530	-7.3004
57.5	-7.7919	-7.6768	-7.5616	-8.0912	-7.5616
62.5	-7.9026	-7.7415	-7.6263	-8.1789	-7.6263
67.5	-8.0001	-8.2994	-7.8619	-8.4606	-7.7007
72.5	-8.2676	-8.2676	-8.1064	-8.4979	-7.9683
77.5	-8.4054	-8.4054	-8.2442	-8.8199	-8.2442
82.5	-8.4591	-8.4591	-8.3900	-8.9656	-8.2288
87.5	-8.7968	-8.4514	-8.3824	-8.7968	-8.3824
92.5	-8.8314	-8.8314	-8.4169	-8.9925	-8.4860
97.5	-8.5282	-8.8735	-8.4591	-9.0347	-8.4591
102.5	-8.9235	-8.9235	-8.5090	-9.0847	-8.5090
107.5	-8.9929	-8.9929	-8.5785	-9.1541	-8.6475
112.5	-9.0708	-9.0708	-8.7254	-9.2320	-8.6563
117.5	-8.8122	-8.8122	-8.5819	-8.8122	-8.5819
122.5	-8.6899	-8.8511	-8.5287	-8.9201	-8.5287
127.5	-8.6474	-8.8086	-8.5093	-8.9698	-8.5093
132.5	-8.6401	-8.7783	-8.6401	-8.7783	-8.6401
137.5	-8.7966	-8.7966	-8.6584	-8.7966	-8.6584
142.5	-8.4402	-8.8317	-8.4402	-8.8317	-8.5554
147.5	-8.4730	-8.6342	-8.3118	-8.6342	-8.4730
152.5	-8.4634	-8.7167	-8.2561	-8.5555	-8.4634
157.5	-8.2828	-8.4209	-8.1446	-8.2828	-8.0065
162.5	-8.0173	-8.1785	-7.8791	-8.4087	-7.7640
b, 1/m	0.0525	0.0697	0.1151	0.0412	0.0465
c, 1/m	0.1614	0.1805	n/a	0.1488	0.1547
b_B, 1/m	0.0011	0.0010	0.0031	0.0009	0.0013
B	0.0210	0.0139	0.0270	0.0217	0.0286
g	0.0100	0.0086	n/a	0.0082	0.0121

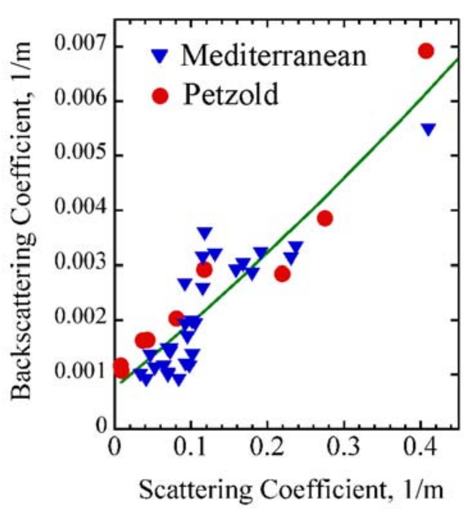

Figure 2. Dependence between backscattering b_B and scattering b coefficients (Fig. 1 zoomed).

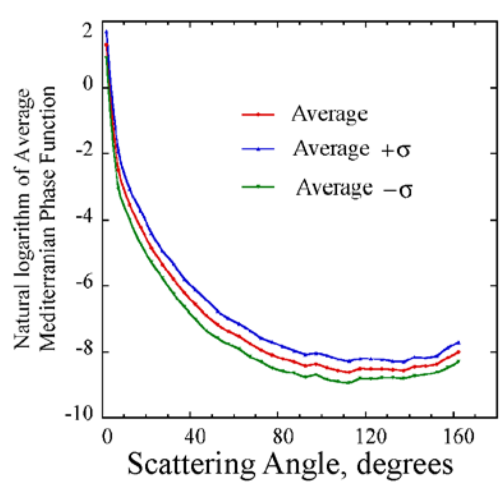

Figure 3. An average Mediterranean phase function with the range of variability determined by the standard deviation σ.

Table 8. Natural logarithms of experimental Mediterranean phase functions of light scattering by sea water $\ln[\beta(\vartheta)/b_0]$ No. 26-30

ϑ \ PhF #	26	27	28	29	30
2.0	0.9506	0.4440	1.6413	1.5492	1.2959
7.5	-2.5658	-3.1644	-2.3355	-2.1513	-2.6118
12.5	-3.8100	-4.2475	-3.2804	-3.4186	-3.6487
17.5	-4.3562	-4.9318	-3.8266	-3.9187	-4.3560
22.5	-4.8749	-5.5887	-4.5296	-4.6217	-4.9899
27.5	-5.4009	-5.9766	-4.8944	-5.2628	-5.4007
32.5	-5.7329	-6.5388	-5.2494	-5.4336	-5.8248
37.5	-6.1837	-6.8515	-5.8153	-5.9535	-6.3217
42.5	-6.4940	-7.1618	-6.0796	-6.3789	-6.4940
47.5	-6.7981	-7.5349	-6.2224	-6.5218	-6.9361
52.5	-7.1392	-7.7379	-6.5866	-6.8629	-7.1392
57.5	-7.2393	-7.9531	-7.0781	-7.1472	-7.4003
62.5	-7.5112	-8.0408	-7.0967	-7.1888	-7.6262
67.5	-7.7007	-8.2994	-7.3093	-7.5856	-7.7005
72.5	-7.8301	-8.4288	-7.4387	-7.8301	-7.9681
77.5	-8.0830	-8.8199	-7.6455	-7.9449	-8.0830
82.5	-8.3900	-8.9656	-7.7913	-7.9295	-8.2287
87.5	-8.3824	-8.9580	-8.0600	-8.0600	-8.3823
92.5	-8.2557	-8.9925	-8.0945	-8.2557	-8.4169
97.5	-8.4591	-8.8735	-7.9986	-8.2979	-8.2978
102.5	-8.5090	-9.0847	-8.0485	-8.1867	-8.5090
107.5	-8.5785	-8.9929	-8.1180	-8.2561	-8.5783
112.5	-8.4951	-9.0708	-8.1958	-8.4951	-8.6561
117.5	-8.7431	-8.8122	-8.1444	-8.2826	-8.5817
122.5	-8.5287	-8.8511	-8.2524	-8.3906	-8.6897
127.5	-8.5093	-8.8086	-8.2099	-8.5093	-8.6474
132.5	-8.6401	-8.7783	-8.2257	-8.3408	-8.5018
137.5	-8.6584	-8.7966	-8.2670	-8.2670	-8.6584
142.5	-8.5554	-8.6705	-8.1179	-8.1179	-8.5553
147.5	-8.4730	-8.7493	-8.2197	-8.3118	-8.4728
152.5	-8.3943	-8.5555	-8.1180	-8.1180	-8.3941
157.5	-8.0065	-8.1446	-7.8913	-7.7762	-8.1445
162.5	-7.7640	-8.0173	-7.6489	-7.7640	-8.0171
b, 1/m	0.1313	0.0336	0.1052	0.0952	0.1681
c, 1/m	0.1520	0.1403	0.2201	0.2089	0.2372
b_B, 1/m	0.0032	0.0010	0.0019	0.0017	0.0030
B	0.0242	0.0294	0.0182	0.0175	0.0179
g	0.1327	0.0092	0.0164	0.0144	0.0416

Table 9. Natural logarithms of Mediterranean phase functions No. 31-32 $\ln[\beta(\vartheta)/b_0]$, $\langle\ln[\beta(\vartheta)/b_0]\rangle$, $\langle\ln[\beta(\vartheta)/b_0]\rangle+\sigma$, and $\langle\ln[\beta(\vartheta)/b_0]\rangle-\sigma$

ϑ \ PhF #	31	32	Average	Aver+ σ	Aver- σ
2.0	2.4242	1.2959	1.2866	1.7033	0.8699
7.5	-1.0230	-2.5658	-2.5082	-1.9312	-3.0852
12.5	-2.6587	-3.6488	-3.5348	-3.0772	-3.9925
17.5	-3.0207	-4.3562	-4.2159	-3.7084	-4.7233
22.5	-3.6776	-4.9901	-4.8490	-4.4055	-5.2926
27.5	-4.0194	-5.4930	-5.3491	-4.9326	-5.7657
32.5	-4.3514	-5.8250	-5.7977	-5.3426	-6.2528
37.5	-4.6870	-6.3219	-6.2176	-5.7948	-6.6403
42.5	-5.0204	-6.3789	-6.5595	-6.1135	-7.0055
47.5	-5.4165	-6.9362	-6.8866	-6.4225	-7.3506
52.5	-5.8037	-7.1392	-7.1580	-6.7467	-7.5693
57.5	-5.9959	-7.4004	-7.3688	-6.9791	-7.7584
62.5	-6.2217	-7.6263	-7.5220	-7.1400	-7.9040
67.5	-6.4343	-7.8619	-7.7432	-7.3459	-8.1404
72.5	-6.6788	-7.8301	-7.9374	-7.5770	-8.2977
77.5	-6.7936	-8.0830	-8.0837	-7.6922	-8.4753
82.5	-6.9854	-8.2288	-8.2044	-7.8334	-8.5753
87.5	-7.0699	-8.2212	-8.2960	-7.9507	-8.6412
92.5	-7.4268	-8.2557	-8.4165	-8.0696	-8.7635
97.5	-7.3078	-8.1367	-8.3620	-8.0323	-8.6916
102.5	-7.5189	-8.3478	-8.4669	-8.1021	-8.8317
107.5	-7.7035	-8.5785	-8.5544	-8.2139	-8.8949
112.5	-7.7813	-8.6563	-8.6063	-8.2755	-8.9370
117.5	-7.5918	-8.5819	-8.5053	-8.2033	-8.8072
122.5	-7.5386	-8.5287	-8.5100	-8.2043	-8.8157
127.5	-7.6573	-8.5093	-8.5075	-8.2260	-8.7890
132.5	-7.7882	-8.5020	-8.5300	-8.2775	-8.7825
137.5	-7.8525	-8.4973	-8.5505	-8.2945	-8.8065
142.5	-7.6804	-8.4402	-8.4403	-8.1544	-8.7261
147.5	-7.5980	-8.3118	-8.4241	-8.1672	-8.6810
152.5	-7.4733	-8.3943	-8.3627	-8.1098	-8.6155
157.5	-7.4769	-8.1446	-8.1734	-7.8734	-8.4734
162.5	-7.1193	-7.8791	-7.9971	-7.6987	-8.2956
b, 1/m	0.2367	0.0731	0.1159	0.1882	0.0437
c, 1/m	0.3667	0.1843	0.2094	0.2861	0.1327
b_B, 1/m	0.0033	0.0014	0.0021	0.0031	0.0010
B	0.0140	0.0198	0.0190	0.0243	0.0137
g	0.0248	0.0128	0.0216	0.0308	0.0113

For the practical purposes it is useful to find eigenvectors of covariant matrix derived from all 32 Mediterranean phase functions. Such analysis was accomplished and the first 5 eigenvectors are presented in Table 10. If we restrict ourselves to 5% precision, the arbitrary Mediterranean phase function may be represented through its average value $\langle\ln[\beta(\vartheta)/b_0]\rangle$ given in the fourth column of Table 9 and the first eigenvector $\psi_1(\vartheta)$ given in Table 10:

$$\beta(\vartheta,\alpha) = b_0 \exp\left\{\langle\ln[\beta(\vartheta)/b_0]\rangle + \alpha\psi_1(\vartheta)\right\} \qquad (2)$$

here α is an arbitrary coefficient.

4. CONCLUSIONS

Presented here thirty two new experimental Mediterranean phase function represent an addition to the Petzold phase functions [1] for the ocean area outside California Bay. These phase functions fit in the same hydro-optical category as offshore Petzold phase functions (see Fig. 2). These phase functions allow us to obtain useful relationship (1) between beam attenuation coefficient and backscattering coefficient. The tables of these phase functions together with accompanied set of related optical properties could be used for radiative transfer modeling using analytical [7, 8] or numerical [9, 10] approaches.

Table 10. Five first eigenvectors of the 32 Mediterranean Phase functions.

ϑ, °	$\psi_1(\vartheta)$	$\psi_2(\vartheta)$	$\psi_3(\vartheta)$	$\psi_4(\vartheta)$	$\psi_5(\vartheta)$
2.0	-0.3453	0.1596	-0.7288	-0.0495	0.0732
7.5	-0.2498	0.2567	0.1654	-0.0163	-0.0705
12.5	0.2098	0.1942	0.2559	-0.0537	0.0768
17.5	0.0648	0.2387	0.1968	0.0724	-0.0930
22.5	0.1039	0.2107	0.1517	0.2825	0.0753
27.5	0.2266	0.2017	-0.0240	-0.3561	-0.2059
32.5	0.0827	0.2236	-0.0955	-0.3045	0.0210
37.5	0.2935	0.2024	-0.1074	-0.0189	-0.0876
42.5	0.0927	0.2159	-0.1546	-0.0250	-0.0844
47.5	0.1901	0.2261	-0.0845	0.2551	0.2721
52.5	-0.0172	0.1995	-0.1361	0.3520	-0.1310
57.5	-0.0584	0.1864	-0.0671	-0.0454	-0.0661
62.5	0.1313	0.1827	-0.0471	-0.0598	-0.2097
67.5	0.1975	0.1867	0.1569	-0.1651	0.3181
72.5	0.0557	0.1692	-0.0258	-0.1439	-0.0218
77.5	0.0824	0.1890	-0.0506	0.1324	0.1100
82.5	0.0311	0.1785	-0.0927	0.0910	-0.3929
87.5	0.0677	0.1626	-0.1560	-0.0577	0.4252
92.5	-0.0360	0.1602	0.1611	0.0265	-0.1889
97.5	-0.1500	0.1534	0.0789	0.0274	0.0722
102.5	-0.4353	0.1664	0.1203	-0.1807	-0.0508
107.5	-0.2692	0.1612	0.0973	-0.0443	0.1540
112.5	-0.2179	0.1535	0.2600	0.1105	-0.0288
117.5	-0.2321	0.1396	0.0245	0.2863	-0.0512
122.5	-0.1433	0.1438	0.1060	0.1037	0.3226
127.5	-0.1956	0.1282	0.1154	-0.1361	-0.1595
132.5	-0.0683	0.1176	-0.0014	-0.0576	0.2097
137.5	-0.0609	0.1186	0.0410	-0.0676	-0.1525
142.5	0.0026	0.1261	-0.0224	-0.3899	0.0544
147.5	0.0902	0.1124	-0.1047	0.2667	-0.0088
152.5	0.1140	0.1101	-0.0840	0.1422	-0.2141
157.5	-0.0106	0.1011	0.0179	-0.0488	0.0831
162.5	0.1394	0.1139	-0.0812	0.0630	0.0133

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6. REFERENCES

[1] Petzold, T. J., Volume Scattering Functions for Selected Ocean Waters, SIO Ref. 72-78, Scripps Institute of Oceanography, Visibility Laboratory, San Diego, CA, 1972, 79 p.

[2] V. I. Haltrin, "Theoretical and empirical phase functions for Monte Carlo calculations of light scattering in seawater," in *Proceedings of the Fourth International Conference Remote Sensing for Marine and Coastal Environments*, I, pp. 509-518, (Envir. Res. Inst. of Michigan, Ann Arbor, MI, USA, 1997).

[3] V. I. Mankovsky, and V. I. Haltrin, "Phase Functions of Light Scattering Measured in Waters of World Ocean and Lake Baykal," - 2002 IEEE International Geoscience and Remote Sensing Symposium and the 24th Canadian Symposium on Remote Sensing Proceedings on CD ROM, June 24-28, 2002, Toronto, Canada, Library of Congress Number: 2002 105858, Paper # I2E09_1759.

[4] V. I. Haltrin, and V. I. Mankovsky, "Analytical Representation of Experimental Light Scattering Phase Functions Measured in Seas, Oceans and Lake Baykal," - 2002 IEEE International Geoscience and Remote Sensing Symposium and the 24th Canadian Symposium on Remote Sensing Proceedings on CD ROM, June 24-28, 2002, Toronto, Canada, Library of Congress Number: 2002 105858, Paper # I1E09_1287.

[5] V. I. Mankovsky, and M. N. Kaigorodov, "Laboratory auto-collimated transmissometer with variable base," pp. 91-92 in *Automatizaton of Scientific Investigations of Seas and Oceans*, (Marine Hydrophysical Institute Ukrainian Academy of sciences, Sevastopol, Crimea, 1980), in Russian.

[6] V. I. Mankovsky (a.k.a V. I. Man'kovsky), "Marine pulse-time nephelometer," pp.81-89 in *The Probes for Scientific Investigations and Automatic Systems*, (Naukova Dumka, Kiev, Ukraine, 1981), in Russian.

[7] V. I. Haltrin, "Self-consistent approach to the solution of the light transfer problem for irradiances in marine waters with arbitrary turbidity, depth and surface illumination," *Appl. Optics*, **37**, 3773-3784 (1998).

[8] V. I. Haltrin, "Apparent optical properties of the sea illuminated by Sun and sky: case of the optically deep sea," *Appl. Optics*, **37**, 8336-8340, (1998).

[9] C. D. Mobley,and L. K. Sundman, *Hudrolight 4.1 User Guide,* Sequoia Scientific, Inc., Redmond, WA, 85 pp. (2000).

[10] C. D. Mobley, *Light and Water* (Academic Press, San Diego–Toronto, 1994).

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